QUANTIFYING THE PROBABILITY OF DEFAULT AS ASSESSED BY THE BOND MARKET: AN ANALYSIS OF DEFAULT RISK MEASURES

BY

MARY STEARNS BROSKE

A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL
OF THE UNIVERSITY OF FLORIDA IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1982

To my parents, Mr. and Mrs. George L. Stearns, Sr., in gratitude for their unfailing support, confidence and encouragement throughout my life, and to my husband, Ernest, and daughter, Elizabeth, in gratitude for their support and many sacrifices throughout the course of this study.

ACKNOWLEDGMENTS

The guidance and direction of Professor Haim Levy, chairman of my dissertation committee, and the many helpful comments of the other members, Professor Moshe Ben-Horim and Professor Rashad Abdel-khalik, are gratefully acknowledged. In addition, financial support for this research from the Center for Econometrics and Decision Sciences, University of Florida, is gratefully acknowledged.

TABLE OF CONTENTS

PAGE

ACKNOWLEDGMENTS iii

LIST OF TABLES vi

LIST OF FIGURES viii

ABSTRACT ix

CHAPTER

I INTRODUCTION 1

II REVIEW OF THE LITERATURE 6

Decomposition of Bond Yields and of Bond Prices 6
Bond Risk and Return: CAPM and Bonds 10
Bond Ratings and Bond Risk 15
Prediction of Bankruptcy and of Default 23
Conventional Ranking Techniques: A Review 26

III PRESENTATION OF THE MODEL 32

Introduction 32
The Definition of Default 36
Measuring the Probability of Default 37
An Algorithm for Calculating the Probability of Default When There Are N Intersections Between the Cumulative Probability Distributions 43
Numerical Examples 47

IV METHODOLOGY AND RESULTS 50

Introduction 50
Methodology 52
Description of Data and Presentation of Empirical Results 56
Analysis of Results 62

CHAPTER PAGE

V SUMMARY AND CONCLUSIONS 82

Introduction. 82
Summary of Results. 83
Proposals for Future Research 90
Portfolio Implications Research. 90
Efficient Set Research 92
Bond Market Efficiency Research. 92
Early Warning System Research. 93
Bond Rating Accuracy Research. 94
Other Research 94
Conclusions 95

APPENDICES

A A COMPARISON OF MOODY'S AND S&P'S CORPORATE BOND RATING DESCRIPTIONS 96

B A COMPARISON OF MOODY'S AND S&P'S CORPORATE BOND RATING DETERMINATIONS 100

C REVIEW OF STOCHASTIC DOMINANCE RULES. 104

D SIX MONTH TOTAL HOLDING PERIOD RETURNS CALCULATED FOR PORTFOLIOS. 108

E CHANGES IN UNADJUSTED GNP USED TO CONVERT NOMINAL SIX MONTH TOTAL HOLDING PERIOD RETURNS TO REAL SIX MONTH TOTAL HOLDING PERIOD RETURNS 113

REFERENCES 115

BIOGRAPHICAL SKETCH. 121

LIST OF TABLES

TABLE		PAGE
1	Bond Ratings and Default Experience: 1900-1943.	25
2	Bond Rating and Default Experience by Decade: 1920-1939.	25
3	Relative Probabilities of Default of Corporate Bonds When the Investment Horizon is Six Months (nominal data).	60
4	Relative Probabilities of Default of Corporate Bonds When the Investment Horizon is Six Months (real data).	60
5	Relative Probabilities of Default of Corporate Bonds and Government Bonds When the Investment Horizon is Ten Years.	61
6	Relative Probabilities of Default of Corporate Bonds When the Investment Horizon is Twenty Years (1971-1980).	63
7	Relative Probabilities of Default of Corporate Bonds When the Investment Horizon is Twenty Years (1969-1980).	63
8	Additivity of Relative Probabilities of Default as Demonstrated by Table 5 Data	68
9	The Impact of the Investment Horizon on the Magnitude of the Relative Probability of Default of Baa Corporate Bonds Over Aaa Corporate Bonds.	72
10	Analyze Yield-to-Maturity Data for Sensitivity of Delta to Change in D_1.	76

TABLE PAGE

11 Analysis of the Variable D_1 for Data Used in Calculating Delta as Presented in Table 7 and in Table 10 78

12 Statistical Analysis of the Distributions of D_1 in Order to Determine Whether Table 11(a) is Not Significantly Different From Table 11(b) 80

13 Values of Statistical Tests for Equality of Means and of Variances Within Bond Rating Categories When D_1 is Calculated for Bonds Grouped in Portfolios and for Individual Bonds. 80

LIST OF FIGURES

FIGURE		PAGE
1	A Comparison of Two Securities: The Treynor Index.	30
2	A Comparison of Two Securities: The Sharpe Index	30
3	Shift in Aaa Distribution (Case 1: Default Results in Zero Return)	41
4	Shift in Aaa Distribution (Case 2: Default Results in Return Less Than Promised Return).	41
5	Cumulative Probability Distributions for Example 2 Alternatives C and D Demonstrating Shift in D to D' When Implied Delta is Included in D	49

Abstract of Dissertation Presented to the Graduate Council of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

QUANTIFYING THE PROBABILITY OF DEFAULT AS ASSESSED BY THE BOND MARKET: AN ANALYSIS OF DEFAULT RISK MEASURES

By

Mary Stearns Broske

August, 1982

Chairman: Professor Haim Levy
Major Department: Finance, Insurance and Real Estate

Bond rating agencies classify corporate bonds and state and municipal bonds into different categories according to the agencies' assessment that the issuer will default on the bond prior to maturity. Bond ratings rank bonds according to the agencies' assessment of the bonds' relative probabilities of default. If the market is efficient and is in equilibrium, the market price of a bond is a function of the risk incurred by ownership of that bond. There are two types of risk inherent in bonds: (a) interest rate risk (variability risk) and (b) default risk.

This study proposes a technique for quantifying default risk (type (b) risk) as estimated by the bond market. The technique developed involves comparing cumulative probability distributions of rates of return (or of yields-to-maturity) of bonds which differ only in default risk once interest

rate risk is neutralized. The empirical observation that corporate bonds (say Aaa) dominate government bonds by First Degree or by Second Degree Stochastic Dominance cannot hold in equilibrium, for Aaa bonds possess default risk and government bonds do not. The technique proposed in this paper involves systematically changing the Aaa bond distribution until the dominance disappears. This inclusion of the implied probability of default results in quantifying the probability of default as assessed by the bond market.

The probabilities of default as presented in this study were consistently in the direction predicted by economic theory, as follows: (1) the magnitude of default probability varied inversely with the quality of the bond (indicated by the bond rating), (2) the magnitude of default probability for a given rating category was consistently larger in periods of economic contraction than in periods of economic expansion, and (3) the magnitude of default probability strictly increased with the length of the investment horizon.

CHAPTER I
INTRODUCTION

The required return on any risky asset contains a premium to compensate the holder of that asset for the risk incurred by the ownership of the asset. The estimation of the risk premium has been a subject of research in both economics and finance, and is of interest in theory and in practice.

There exist several commonly accepted and widely used models of the relationship between the expected return (in equilibrium, this is identical with the required return) and the risk inherent in the ownership of risky assets. This body of literature evolved from the study of stocks and how they are priced. By extension, existing valuation theory is assumed to apply to all risky assets. The Capital Asset Pricing Model, or CAPM (Sharpe, 1964; Lintner, 1965; Mossin, 1966), suggests that the relationship between risk and expected return is linear, and that only systematic risk, and not total risk (variance), is relevant to investors. Assets are priced so as to compensate investors for this systematic risk. Since the remaining portion of total risk is diversifiable risk, and the marginal investor is assumed to hold a diversified portfolio, the expected return contains

only a premium for the non-diversifiable risk. The CAPM assumes that investors have von Neumann-Morgenstern utility functions, and are concerned with the first two moments only and that the distributions are normal. Empirical tests of the CAPM have produced mixed results. In general, when the CAPM is tested on stocks, the relationship is linear but the intercept is higher and the slope is less than theory predicts.

The Arbitrage Pricing Model, or APM (Ross, 1976), represents an attempt to avoid Roll's (1977) criticisms of the CAPM in that the market portfolio plays no part in determining the expected return on a given risky asset. Rather, the expected return is a linear function of economy-wide factors, the levels of which are estimated by well-diversified investors, times a beta-factor for the sensitivity of the security to that factor. Ross himself suggests that validation of the APM is empirically intractable since it is not possible to identify all the factors.

Stock valuation models are even less accurate when used to evaluate risky bonds. This suggests that risk for bonds may be multi-dimensional rather than linear in nature. The measures of bond risk suggested in the literature are unidimensional in that they are assumed to be of equal significance for all risky bonds. The horizon (the holding period) is of significance in bond valuation and implies that any attempt to develop one risk measure (or one group or cluster of risk measures) appropriate for bonds of different

maturities held for different holding periods may be inappropriate. There are two types of risk inherent in bonds: (a) variability risk and (b) default risk.

Bond rating agencies provide an estimate of the quality of most large, publicly held corporate and municipal and state governmental bond issues by assigning ratings to these issues, and by revising these ratings when the agency's estimate of the quality changes. Bond ratings are assigned primarily on the basis of the agency's assessment of the probability that the firm will default, and are intended to rank bond issues according to their relative probabilities of default. Rating agencies state that there is no set formula for determining the rating, but rather that all available information about the firm and the issue are considered. The relationship of the bond rating to the probability of default has been analyzed in the literature. In general, the quality of bonds as indicated by bond ratings has been determined to be inversely related to the probability of default as indicated by the frequency of default.

The purpose of this paper is to estimate the probability of default as assessed by the bond market. In order to quantify the relative probabilities of default of bond rating categories, a stochastic dominance measure of the probability of default is derived. The measure is applied to distributions of yields-to-maturity and of holding period returns for investment grade corporate bonds and for government bonds, and the resulting relative probabilities of

default are analyzed. The stochastic dominance measure developed in this study is proposed as a technique for analyzing the manner in which the bond market estimates relative probabilities of default.

Chapter II presents an overview of the literature concerned with bond risk and its measurement. Both those studies which developed predictors of default and those which analyzed bond ratings as predictors of default are reviewed in this section.

Chapter III includes the development of the model, some simple examples to illustrate its application, and the derivation of an algorithm for implementing the model in the general case.

Chapter IV presents the methodology employed and the empirical results. It includes a description of the data followed by an analysis of the data for the full period of the study and for sub-periods of economic expansion and of economic contraction.

Chapter V presents the summary of the study and the conclusions to be drawn from it. In general, the results of the empirical work indicate that the stochastic dominance technique developed in this study has economic validity. The resulting estimates of the relative probabilities of default were consistently in the right direction in that the estimates were correctly related to the level of economic activity. The estimates were in general larger in periods

of economic expansion than they were in periods of economic contraction. In addition, the magnitude of the estimates increased with the length of the investment horizon, as one would expect.

CHAPTER II
REVIEW OF THE LITERATURE

Decomposition of Bond Yields and of Bond Prices

Economic theory suggests that the yield on a bond, as on any risky asset, should consist of the real rate of interest, a premium to compensate for expected inflation, and a risk premium to compensate for any other non-diversifiable risk inherent in ownership of the asset (Hicks, 1939; Keynes, 1930; Lutz, 1940). In general, there are four suggested components of the yield on a bond: (1) the pure, or certain, rate of interest which reflects the underlying dynamics of the economy; (2) a premium for credit risk, which is the risk of defaulting on either the payment of interest or principal; (3) a premium for purchasing power risk, which is the risk of a decline in the purchasing power of the interest and principal payments; and (4) a premium for interest rate risk, which is the risk of an increase in the market rate while the bond is held (Levy and Sarnat, 1972, pp. 104-114; Foster, 1978, pp. 444-445). These are not derived from a developed theory of bond pricing, but rather they have been developed to explain empirically observed differences in yields of different bonds at a

point in time, or in yields of the same bond at different points in time (Foster, 1978, pp. 444-445).

The classic empirical work on the determinants of the bond risk premium is that of Fisher (1959). He defined the risk premium as the difference in yield between a corporate bond and a government security of the same term to maturity. The risk premium was hypothesized to have two components--a default premium and a marketability premium. The risk of default was associated with three variables: (1) the coefficient of variation of the firm's earnings (net income) over the last nine years; (2) the length of time that the firm has operated without creditors having suffered a loss; and (3) the ratio of the market value of equity to the par value of debt. The marketability of the bond was estimated by the market value of all the publicly traded bonds that the firm had outstanding. This was assumed to proxy transaction frequency.

Fisher found that the logarithms of the four variables accounted for approximately 75 percent of the variance in the logarithm of the risk premium. Unlike others, he found that the logarithm regression coefficients were relatively stable over time. As is often the case with earlier empiricists, the model that Fisher developed was ad hoc in nature. The variables were selected based on Fisher's hypothesis that they had explanatory power, rather than selected according to any existing theory of the components of the risk premium.

Hastie (1972) employed Fisher's methodology in studying municipal bond yields. He also suggested the existence of a default premium and a marketability premium, but he estimated them differently. The default premium was estimated by (1) the ratio of overall debt to true property values, (2) default history, (3) economic diversification, and (4) college students as a percentage of the issuer's population. The marketability premium was estimated by (1) the size of the block offered, (2) the net debt of the issuer, and (3) the past population growth. He found that the relative significance of the variables depended on whether commercial banks or individuals dominated the market. His regression coefficients explained about 86 percent of the variability in the risk premium, but the coefficients were not stable over time and were sample-sensitive at a given point in time.

Hastie's model is also ad hoc in nature. Although he suggests that his inclusion of a default premium and a marketability premium is theoretically based, a closer analysis reveals that his work is based on his hypotheses about what the rational investor would like or dislike. There is no explanation offered as to why some variables are selected and other possibilities are omitted.

An alternative approach to bond valuation is exemplified by Silvers (1973) who investigated the determinants of the bond price, rather than the risk premium. He tested the relationship between bond price and the following independent

variables: (1) a vector of coupon certainty equivalent coefficients, (2) a call variable, and (3) a marketability measure. He sorted his sample by bond rating category, and concluded that all the independent variables except marketability were significant.

A current study by Boardman and McEnally (1981) readdresses the issue of the bond yield (or premium) which concerned Fisher and Hastie, and the issue of bond price determinants which was addressed by Silvers. The study decomposes corporate bond prices into components representing (1) the pure price of time, (2) the default risk of the bond's agency rating class, and (3) bond-unique risk. The authors, however, in analyzing bond price determinants, do so by adding variables so that up to 45 variables enter into each of the 16 versions of the price equation. This study is also ad hoc in that variables are added because they are thought to be of relevance in predicting bond price. The study is neither based on an existing theory of bond valuation nor does it attempt to develop such a theory. Rather, in the absence of theory, it attempts to analyze the significance of a large number of variables which were included either in order to replicate earlier studies or in order to add variables the authors thought missing in previous work.

Thus, attempts to identify the components of the risk premium or the components of bond prices in general seem to involve researchers selecting a set of variables which are

thought to be significant, then testing to see if they are indeed significant. All such studies are subject to the criticism that they are not based on theory. Even though it is common for an author to state economic interest rate theory as the basis for his research, a closer analysis indicates that the selection of the variables to be used is ad hoc.

Bond Risk and Return: CAPM and Bonds

There is not a well-developed theory of bond pricing under uncertainty which can be said to correspond to the CAPM. In theory, since the CAPM is an expression of the risk and return relationship for any risky asset, it is applicable to bonds although it is more commonly applied to stocks.

The linear relationship between risk and return which is expressed by the CAPM has been tested both on stocks (for example, Douglas, 1969; Lintner, 1965; Miller and Scholes, 1972; and Black, Jensen and Scholes, 1972) and on bonds (Percival [1974] did the initial work, followed by Reilly and Joehnk [1976], and Friend, Westerfield and Granito [1978]).

In general, the empirical tests using stock data show a linear relationship, although the intercept is higher than the observed risk-free rate and the slope is less than theory would predict. In addition, there is some evidence that the standard deviation in some cases, or the residual variance

in others, is significant. Thus, in general, the theoretical model does not explain the empirical evidence in a satisfactory manner.

When the CAPM is applied to bonds, the results are even weaker, as a survey of the literature indicates. The initial attempt to apply the CAPM to bonds was done by Percival (1974) for the period 1953-1967. Betas were estimated over the full period, and realized returns were calculated as the geometric mean holding-period yields. When the regression

$$\bar{R}_i = a + b\hat{\beta}_i + e_i$$

was run, the regression results were as follows: $R^2 = .15$, $a = .0389$, $b = -.00931$. The t-value was -5.629. Percival explained the negative sign as resulting from generally rising interest rates during the period (Percival, 1974, p. 464). He then added dummy variables for industry and for bond rating. The regression results then were $R^2 = .4680$, $a = .0370$, and $b = .00983$. The b-coefficient and the bond rating coefficients were all significant at the $\alpha \geq .05$ level. Finally, he analyzed beta as a function of the industry, the rating, the coupon rate, and the term to maturity, getting $R^2 = .3345$, with intercept = 1.19605. The significant variables at $\alpha \geq .05$ level were railroad industry, coupon rate, and maturity (Percival, 1974, p. 464).

Percival concluded that bond betas are measures of interest rate risk, but that they must be combined with a

non-diversifiable default-risk measure in order to explain realized bond returns (Percival, 1974, pp. 464-468).

Reilly and Joehnk (1976) assumed that a market-derived beta for a bond should be an appropriate risk measure for bonds, and that this beta should be inversely related to bond ratings. Since they assumed that the CAPM should relate to bonds, they did not actually test it. Their resulting study of bond ratings and risk measures, however, provides information about the use of the CAPM on corporate bonds. The CAPM assumes that beta risk is the relevant risk, whereas Reilly and Joehnk's results indicate that relevant risk for bonds is total risk (σ_i^2). They found that the bond rating contained useful information for the pricing of bonds, and as bond rating was a better indicator of total risk than of market risk, then by extension, total risk has importance for bond pricing.

Yawitz and Marshall (1977) applied the CAPM to the government bond market. They used the excess return form of the CAPM, and found in every case that the intercept was positive and statistically significant, and that both beta and sigma had about equal explanatory power. (They were not included in the same regression at the same time.) R^2 ranged from about 63 and 77 when beta was used to about 70 and 76 when sigma was used as the risk measure (Yawitz and Marshall, 1977, p. 20). Their finding of the equivalent strength of market risk and total dispersion of return (σ) as a risk measure indicates that the CAPM is not satisfactory for bonds.

Finally, Friend, Westerfield and Granito (1978) tested the CAPM on corporate bond returns as part of a comprehensive re-testing of the CAPM by incorporating bonds into the market index. The Sharpe-Lintner version of the CAPM yielded the following regression results for corporate bonds: $\bar{R}^2 = -.001$, $a = 1.016$, $b = .001$. When a second regression was run, including both beta and the standard deviation of the residual term, the results were as follows: $\bar{R}^2 = .013$, $a = 1.017$, $b = .001$. None of the risk measures was statistically significant for individual bonds (Friend, Westerfield and Granito, 1978, p. 910). It should be noted that they did not use bond ratings as an explanatory variable. Hence, in conclusion, none of the tests of the CAPM using bond data has validated the use of that model in explaining the relevant risk for bonds. Rather, the evidence seems to indicate that market risk is not the only relevant risk for investors in bonds. The relationship of risk to return in bonds does not seem to be linear. Historical studies of bonds have always indicated a non-linear relation. McCallum's (1975) study of Canadian government bonds (1948-1968) indicates that the total holding period return was a non-linear function of maturity, standard deviation, and beta (McCallum, 1975, Tables 1, 2, 3).

Further work needs to be done on the nature of risk for bonds, especially for bonds of different quality. None of the above studies separated bonds by rating, for instance,

before applying the CAPM. Such a procedure would indicate whether the intercept, slope(s), significance, and R^2's are different for different grades of bonds. This would indicate whether the nature of risk is different for different bonds.

Recent studies suggest that the estimates of the CAPM's beta depend on the assumed investment horizon, as systematic risk is a function of the length of the horizon for stocks (Levy, 1981), and for stocks and bonds (Kaufman, 1980). These authors conclude that the failure of prior tests of the CAPM on stock data and on bonds may be explained by the omission of the investment horizon. Kaufman offers this as the explanation of why the CAPM is more successful in pricing equities than it is in pricing bonds (Kaufman, 1980, p. 1). Levhari and Levy's (1978) finding that the systematic risk of aggressive stocks ($\beta_j > 1$) increases with the investment horizon and the systematic risk of defensive stocks ($\beta_j < 1$) decreases with the horizon led Levy to re-examine the results of the classic empirical tests of the CAPM in order to see whether the effect of the assumed investment horizon could explain the documented poor results (Levy, 1981, p. 37). The results were not much different from those of the earlier tests, but the empirical results varied with the assumed horizon (Levy, 1981, p. 38). These studies cited suggest that it is necessary to consider the investment horizon when analyzing risk and return for stocks and, perhaps even more importantly, for bonds.

As it is the purpose of this paper to analyze existing risk measures and to apply Stochastic Dominance Criteria in quantifying default risk, then it is appropriate to consider bond ratings as they relate to bond risk.

Bond Ratings and Bond Risk

Bond rating agencies provide an estimate of the quality of most large, publicly held corporate and municipal and state governmental bond issues by assigning ratings to these issues, and by revising the assigned ratings when the agency's estimate of the quality changes. The two primary rating agencies are (1) Moody's and (2) Standard and Poor's. Their rating assignments and descriptions of the rating categories are available by subscription, and are published weekly and monthly. A description of Moody's bond ratings and a comparable description of Standard and Poor's bond ratings are displayed in Appendix A. In addition, bond rating assignments and changes in ratings are reported in The Wall Street Journal.

Bond rating services, such as Moody's and Standard and Poor's, assign ratings to bond issues primarily on the basis of the agency's assessment of the probability that the firm will default. A partial description of the considerations which enter into the bond rating is presented in Appendix B. Bond ratings rank issues in order of the probability of default. Thus, a rating of Aaa (Moody) or AAA (Standard

and Poor) is assigned to bonds having a negligible probability of default. The second category is Aa or AA (Moody and S&P, respectively), followed by A (for both) and Baa or BBB (Moody and S&P, respectively). Bonds rated below this last category are considered speculative, and are not considered to be investment quality. Only issues of the Federal government are assumed to have no risk of default, as the Congress has the authority to issue money to settle its debt.

Bond rating agencies state that there is no set formula for determining the bond rating, but rather that all available information about the firm is considered. However, there seem to be criteria in common use by all bond analysts in setting the rating (Cohen, Zinbarg and Zeikel, 1977, p. 388). These criteria are as follows (pp. 389-390):

1. Protective provisions of the issue
2. Ratio analyses
    a. Fixed charge coverage
    b. Long-term debt to equity
    c. Liquidity position, both current and projected
3. Other considerations
    a. Size and economic significance of the firm
    b. Economic significance of the industry

Perhaps one-half of all bonds are rated identically by different agencies. Where there are differences, they are usually not greater than one category (Cohen, Zinbarg and Zeikel, 1977, p. 385).

The function of bond rating agencies in an efficient market is not clear, for there are many empirical studies which succeed in predicting bond ratings on the basis of publicly available information. Kaplan and Urwitz (1979) provide a thorough survey of statistical models of bond ratings. They then develop and test a linear model using a dummy variable for subordination and variables for total assets, long-term debt to total assets ratio, and the common stock's beta and standard error of residuals. Their model correctly predicted the rating (Moody's) for about two-thirds of the new issues studied, and the errors in prediction were no more than one rating category away from the actual rating category assigned by the bond rating agency. In addition, their results led them to conclude that their model may be predicting the actual riskiness of bonds (based on calculated market yield of these bonds) better than the bond rating prediction of this risk in the case of some of the misclassified bonds (Kaplan and Urwitz, 1979, p. 256). Their finding that market risk (β) is significant in predicting ratings on new issues, whereas unsystematic risk as measured by the estimated standard error of residuals in the market model used to estimate beta is significant in predicting ratings on seasoned issues (issues which have been traded for awhile in the secondary market), lends logical support to an hypothesis that the bond rating itself may be contributing to the magnitude of the risk premium. Perhaps

this is an indication of bond market inefficiency. There is no economic reason that on the average new bond issues should contain a premium for systematic risk, but not for non-systematic risk, whereas seasoned issues should contain a premium for non-systematic risk. Of course, it should be noted that the systematic risk measure is the stock beta and the non-systematic risk measure is the estimated standard error of residuals resulting from estimating the stock beta. Nevertheless, these variables were determined to be significant in predicting bond ratings, with the stock beta having significance for new issues and the estimated standard error of residuals for seasoned issues.

Another approach to the study of bond ratings and bond market efficiency is exemplified by research analyzing bond price changes around the date of bond rating changes. Weinstein (1977), Katz (1974), Grier and Katz (1976), and Hettenhouse and Sartoris (1976) represent this body of literature. Weinstein is the only one of these authors to find some evidence of price changes anticipating rating changes. He found some indication of price changes occurring from eighteen months to seven months prior to the announcement of the rating change. He found no evidence of price change during the six months prior to the rating change, and little or no price reaction during the month of the change or for six months thereafter (Weinstein, 1977, p. 342). The earlier studies cited above found no evidence that the

market anticipates bond ratings changes, and instead report evidence indicating that bond prices adjust to the announcement of a change in rating. Katz (1974), for example, reports that there is on average a lag of six to ten weeks in the price adjustment process (Katz, 1974, p. 558). Weinstein concludes that his findings indicate that the bond market is semi-strong efficient,[1] and that the price change that he found prior to the rating change results from information that eventually leads to a change in the rating. In conclusion, the question of whether the bond market is efficient is not conclusively answered in the literature. Also, the question of whether the bond rating has value in an efficient market, given that bond rating can be predicted to a certain degree, needs further research.

The relationship between bond rating and bond risk has been studied empirically. The initial investigation of the relationship of bond rating to market-determined bond risk was that of Reilly and Joehnk (1976). They estimated several market-derived betas by using various proxies for the market, and selected Moody's Average Corporate Bond Yield Series as the best for their purposes. Capital market theory influenced their hypothesis that the market-derived beta for a bond should be an appropriate risk measure for bonds, and

---

[1]Semi-strong market efficiency implies that no investor can earn excess returns by using trading rules based on publicly available information. The definition appears in most basic finance textbooks, and credit is given to Eugene Fama (1970) for developing and operationalizing the idea.

that this beta should be inversely related to bond ratings (Reilly and Joehnk, 1976, p. 1389). They refer to the empirical relationship between accounting-determined risk measures, based on internal corporate variables, and the market-determined risk measure, β. They cite studies by Beaver, Kettler and Scholes (1970), Logue and Merville (1972), Breen and Lerner (1973), and Gonedes (1973) as documenting the relationship (Reilly and Joehnk, 1976, p. 1388). They reasoned that if bond ratings are related to some of the same corporate variables as are bond betas, then there should be an inverse relationship between bond betas and bond ratings.

They performed several tests of the association between bond betas and bond ratings and hypothesized a significant difference in the betas for different rating classes. Their results did not consistently support the hypothesized significant and negative relationships between bond betas and bond ratings. In general, bond betas for adjacent rating classes (i.e., Aaa versus Aa) were not significantly different. In addition, even when the hypothesized differences were significant, the differences were in the wrong direction. These results remained as stated above even when the rating categories were widely separated (Aaa versus Baa). They concluded that bond betas were not consistently related to bond rating. They did not investigate the possibility that the strength of the relationship between bond betas and

bond ratings may differ from one bond rating category to another.

Reilly and Joehnk in the same paper also investigated the relationship between bond rating and total risk (defined as the standard deviation of monthly percent price changes). The total risk measures for industrial bonds were all in the predicted order. The Aaa measure was the smallest, and each successive rating class average was larger, as predicted. The risk measures for the top three classes were significantly lower than for the lowest class (Baa). Thus, there is an inverse relationship between bond ratings and total risk, as defined by Reilly and Joehnk. However, they did not address the question of whether the strength of this relationship varies with the bond rating category.

Another study which contains information about the relationship between bond ratings and bond risk, as represented by β and by σ, is Friend, Westerfield and Granito (1978). In testing the CAPM based on bond returns, it was necessary for them to estimate beta and sigma so as to rank the bonds by beta (β_i) decile and then by residual standard deviation (σ_{ri}) decile within each beta decile. The expected values of β_i and σ_{ri} were estimated from regressions of these measures of risk on the bond's S&P quality rating, its maturity, and its coupon rate. Although the study did not address the question of the relationship between bond rating and bond risk measures, the results of the regressions

contain useful information about the question. The regression results were as follows:

$$\beta_i = \underset{(11.1)}{.28Q(1)} + \underset{(10.5)}{.23Q(2)} + \underset{(10.6)}{.23Q(3)} + \underset{(6.4)}{.19Q(4)} + \underset{(7.1)}{.30Q(5)} + \underset{(4.8)}{.23Q(6)} + \underset{(11.8)}{.47Q(7)} - \underset{(-.38)}{.06C} + \underset{(7.6)}{.01M}$$

$$\bar{R}^2 = .13$$

$$\sigma_{ri} = \underset{(11.1)}{.03Q(1)} + \underset{(13.5)}{.03Q(2)} + \underset{(14.3)}{.03Q(3)} + \underset{(15.3)}{.05Q(4)} + \underset{(12.6)}{.06Q(5)} + \underset{(17.2)}{.09Q(6)} + \underset{(19.4)}{.08Q(7)} - \underset{(-7.2)}{.12C} + \underset{(5.4)}{.0004M}$$

$$\bar{R}^2 = .33$$

where Q(1) is the highest and Q(7) the lowest S&P quality rating, C is the coupon rate, and M is the years to maturity; the numbers in parentheses are the t-statistics of the regression coefficients, and $\bar{R}^2$ is the coefficient of determination adjusted for degrees of freedom (Friend, Westerfield and Granito, 1978, p. 912).

The coefficients for the bond rating dummy variables are all statistically significant in both regressions. The interesting question is whether they are significantly different from one another in their contributions to the level of the dependent variables. There is an Analysis of Covariance technique discussed in McNeil, Kelly and McNeil (1975, pp. 237-239) for determining whether the group membership (in this case, bond rating category) has additional explanatory power on the level of the dependent variable over and

above any effects attributable to initial differences in the covariables (in this case, coupon rate and years to maturity). This technique is applicable to the above two regressions and would require only that they be re-run without any of the dummy variables being included. The procedure can also be done after removing the dummy variables one at a time in order to isolate the relative contribution of each rating to the dependent variable, holding constant the effect of coupon rate and time to maturity. Such an analysis should indicate the type of risk which is reflected in the different bond rating categories. In this study, sigma is not total risk; rather, sigma is the standard deviation of the residual.

The literature reviewed suggests that the relevant risk for high-grade bonds may differ from the relevant risk for low-grade bonds. Further study is needed to ascertain whether total risk or systematic risk is relevant for bonds, and whether the nature of the relevant risk varies with the bond rating. As this paper is concerned with default risk, it is appropriate to consider the relationship between bond ratings and bankruptcy and between bond ratings and default risk as presented in the literature.

Prediction of Bankruptcy and of Default

The prediction of bankruptcy from publicly available information has also been documented, and is primarily

associated with Altman (1968, 1971 and 1977). Multiple discriminant analysis is utilized to develop a bankruptcy classification model which uses financial statement data and market data. The most recent paper (1977), which is represented as demonstrating significant improvement over earlier models, presents the ZETA bankruptcy identification model. The ZETA model had a range of prediction accuracy from 96 percent one period prior to bankruptcy to 70 percent five annual reporting periods earlier (Altman, Haldeman and Narayan, 1977, p. 50).

The relationship of bond rating to the probability of default has also been analyzed empirically. The classic study of the relationship of bond rating to the frequency of default is that of Hickman (1958). He collected extensive data on default experience for all large outstanding issues during the period 1900 to 1943. His results are summarized in Table 1, and indicate that in the period studied, the probability of default (as indicated by the occurrence of default) is inversely related to the quality of the bond as reflected in the bond rating. Hickman (1960) reported the number of defaults by bond rating broken down by decade for the twenties and thirties and for the two decades together. Table 2 presents Hickman's results as reported by Pye (1974). Pye concluded that Hickman's data indicates that almost all the spread between Aaa and Baa bonds in the twenties and thirties would have to be due to a default premium. He defined the default premium as the

Table 1. Bond Ratings and Default Experience: 1900-1943

Bond Rating	Comparable Moody's Rating	% Defaulting Prior to Maturity
I	Aaa	5.9%
II	Aa	6.0
III	A	13.4
IV	Baa	19.1
V-IX	Below Baa	42.4

Source: W. Braddock Hickman, Corporate Bond Quality and Investor Experience, Princeton, N.J.: Princeton University Press, 1958, Table 1, p. 10.

Table 2. Bond Rating and Default Experience by Decade: 1920-1939

Bond Rating	Period		
	1920-1929	1930-1939	1920-1939
I	.12%	.42%	.3%
II	.17	.44	.3
III	.20	1.94	1.1
IV	.80	3.78	2.3

Source: W. Braddock Hickman, Statistical Measures of Corporate Bond Financing Since 1900 (1960) cited in Pye (1974).

difference between the yield (the coupon rate, or promised return) and the expected return (the yield-to-maturity) (Pye, 1974, p. 49). Pye analyzed default occurrences in the fifties and sixties and found that default experience on investment grade bonds was quite different than for the earlier period studied by Hickman. Pye found virtually no incidence of default in the fifties and sixties for bonds rated Baa or better. Incidentally, he noted that Lockheed was rated Baa in 1960, but did not default because Congress intervened (Pye, 1974, p. 52). Pye concluded that for the post-war period, the probability of default is so small as to be insignificant; thus the premium on low grade bonds when compared with high grade bonds is a risk premium. It is Pye's opinion that the post-war default experience should continue into the future (Pye, 1974, p. 52). This is an empirical question and remains to be answered in relation to the observed differences in bond ratings.

Conventional Ranking Techniques: A Review

Because bond valuation theory is not as well developed as is stock valuation theory, the bond investor has fewer and less sophisticated methods available for making the investment decision than does an individual wishing to invest in stocks. Models for the independent estimation of what the equilibrium price on a stock should be are more accurate than are techniques for the equivalent estimation

of the equilibrium price on a bond. This may be due to the relative lack of quantifiable measures of risk for bonds. As a result, there have evolved techniques which may be used for ranking bonds, thus enabling the bond investor to select the preferred bond, or bonds, from the set under consideration.

The reward-to-risk measures of Jensen, of Treynor, and of Sharpe can be used to rank bonds, thus enabling the investor to select the preferred bond, or bonds, from the set under consideration. These measures are similar, and have in common the goal of reducing the risk-return evaluation of investment performance to a single measure.[2]

Treynor (1965) suggests that the relationship of excess return to nondiversifiable risk is an indication of the performance of either a security or a portfolio of securities. The Treynor Index is stated as follows:

$$\alpha_i = \frac{Ex_i - R_F}{\beta_i}$$

where

Ex_i = the expected return on the security or portfolio

β_i = the systematic risk of the security or portfolio

R_F = the risk-free rate

---

[2]Levy and Sarnat (1972, p. 480) cite Friend and Blume (1970) as providing the formal relationship of the three performance measures we shall consider (Jensen, Treynor, and Sharpe).

This index relates the excess return above the risk-free rate earned by the security or portfolio to its systematic risk, and thereby assumes that the market prices only systematic risk. If one is comparing securities characterized by the risk premium being a function of the systematic risk only, then the Treynor Index has value as a tool for ranking securities.

An alternative to Treynor's index is Sharpe's (1966) reward-to-variability measure, which is as follows:

$$I = \frac{Ex_i - R_F}{\sigma_i}$$

where

σ_i = the standard deviation of returns on the security.

Sharpe's index was intended to be used as an indicator of the performance of portfolios in general and of mutual funds in particular. Since capital market theory presumes that in equilibrium risky assets are priced so as to compensate investors for the asset's systematic risk, but not for its non-systematic risk, Sharpe's index normally is not applied to individual securities or to inefficient portfolios. The denominator, σ_i, is the square root of the variance of returns, or the total risk of the asset. One may justify applying Sharpe's index to individual bonds on the basis that (1) the precise nature of bond risk is not uniformly agreed upon in the literature and (2) it may be

that systematic risk is the only relevant risk for some bonds, whereas non-systematic risk, or perhaps total risk, is significant in the pricing of other bonds.

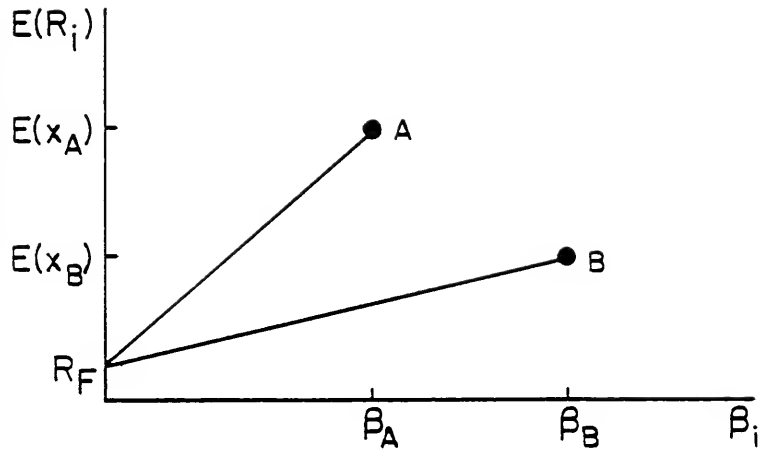
The Treynor index and the Sharpe index are the slopes of transformation lines connecting the risk-free rate and a point representing the risk-return characteristics of the given security. A graphical demonstration of the use of the Treynor index in comparing two securities, A and B, is presented in Figure 1. A comparable demonstration for the Sharpe index is shown in Figure 2. The greater is the magnitude of the index calculated for a given security or portfolio, the greater is the level of expected utility attainable by a risk-averse investor in that security. This interpretation is presented by Levy and Sarnat (1972, p. 482) for Sharpe's reward-to-variability ratio. We apply it to Treynor's index also, as the indexes differ only in the risk measure used in the denominator.

Whereas Treynor and Sharpe devised ratios to indicate the excess return to non-diversifiable risk and the excess return to total risk respectively, Jensen's (1968) Abnormal Performance Index is based on the excess return, and is not a ratio. It is expressed as follows:

$$\alpha_i = (\bar{R}_i - R_f) - \hat{\beta}_i(\bar{R}_m - R_f)$$

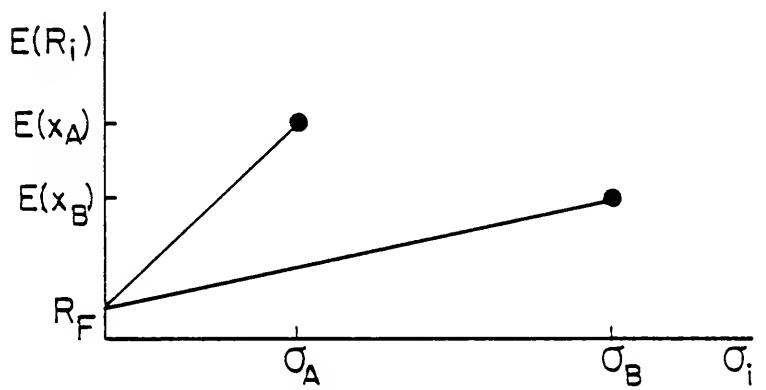
where

$\bar{R}_m$ = the return on the market.



$$\sigma_A = \frac{Ex_A - R_F}{\beta_A}; \quad \sigma_B = \frac{Ex_B - R_F}{\beta_B}$$

Figure 1. A Comparison of Two Securities: The Treynor Index



$$I_A = \frac{Ex_A - R_F}{\sigma_A}; \quad I_B = \frac{Ex_B - R_F}{\sigma_B}$$

Figure 2. A Comparison of Two Securities: The Sharpe Index

Jensen's index is applied to either individual securities or to portfolios of mutual funds, and is based on the Capital Asset Pricing Model. A positive value for α_i indicates that after adjusting for risk, and for movements in the market index, the abnormal performance of the security or portfolio is also positive.

Although these measures are commonly used to rank bonds or portfolios, their dependence on the assumptions of and the validity of the underlying model (the Capital Asset Pricing Model) precludes the inclusion of them as methods of ranking bonds. In addition, the well-documented problem of beta instability, especially as it related to bonds (Weinstein, 1981), adds further validity to the decision not to include these measures as ranking techniques. The techniques examined in this study, bond ratings and the Stochastic Dominance method, are free from dependence on a model, and directly concern themselves with the probability of default. Only the prevalence of the Sharpe, Treynor, and Jensen indexes in practice justifies their inclusion in this survey of ranking techniques.

CHAPTER III
PRESENTATION OF THE MODEL

Introduction

As stated, the purpose of this study is to estimate the probability of default as assessed by the bond market. In order to achieve this goal, the following assumptions are made. Investors, who are assumed to be risk-averters, are faced with the choice of investing $I either in Government bonds or in corporate bonds. For purpose of analysis, assume these corporate bonds are rated Aaa by the bond rating agency. Denote the cumulative probability distribution of the rates of return on an investment in the Government bonds by $F_G(x)$ and the cumulative probability distribution of the rates or return on an investment in the Aaa bonds by $F_{Aaa}(x)$, where x is the rate of return. The expected return on the investment of $I is a function of the risk to which the investor is exposed. As the investor is a risk-averter, he/she will require a higher rate of return for exposure to a higher level of risk. For any given holding period (e.g. one month, one year, etc.), there are two main components included in the risk involved in each investment: (a) risk arising from possible changes in interest rates over the holding period, and (b) risk arising from the probability

that the issuer may default on the bond prior to the end of the holding period. As the purpose of this study is to quantify only default risk, it is necessary to neutralize type (a) risk. This is accomplished by holding maturity (or, ideally duration) constant when comparing the cumulative probability distributions of rates of return or the two types of bonds. This point is discussed further in the presentation of the methodology in Chapter IV.

If type (a) risk is held constant, the risk-averse investor will pay a higher price for the default-free Government bond than for the Aaa corporate bond. The lower price for the Aaa corporate bond implies that the holding period rates of return on this investment will be higher than will be the comparable holding period rates of return on an investment in the Government bond.

On an ex-ante basis, neither $F_G(x)$ nor $F_{Aaa}(x)$ should be expected to dominate the other by Second Degree Stochastic Dominance (SSD) once the default risk is compensated for in the required rate of return. (Stochastic Dominance rules are reviewed in Appendix C.) However, the data consist of ex-post rates of return, and include only bonds of firms which did not default. As a result, on an ex-post basis, $F_{Aaa}(x)$ is expected to dominate $F_G(x)$ by SSD, for an investment in Aaa bonds exposes the investor to default risk, whereas investment in Government bonds does not. There are two possible states (θ) which accompany an investment in corporate bonds:

θ_1 - no default, the state wherein the investor obtains an observation drawn from $F_{Aaa}(x)$ as observed in the past.

θ_2 - default, the state wherein the investor receives either a zero return or some compensation, the magnitude of which depends on the severity of the default.

As this study utilizes ex-post data, it includes only firms which did not default during the period covered by the data. The ex-post data, as they include only firms which did not default, consider only state θ_1, and state θ_2 is not represented explicitly in the data. As stated, ex-post data are expected to reflect the SSD dominance of $F_{Aaa}(x)$ over $F_G(x)$. As a result, it is possible to derive from distribution $F_{Aaa}(x)$ a new distribution $F'_{Aaa}(x)$ which assigns some probability to the occurrence of state θ_2. We can change the magnitude of this probability until neither $F_G(x)$ nor $F'_{Aaa}(x)$ dominates the other by SSD. This probability of state θ_2 which results in neither distribution dominating the other by SSD is the risk of default of the Aaa bond as assessed by the bond market. Assuming that the market is efficient and is in equilibrium, neither $F_G(x)$ nor $F'_{Aaa}(x)$ is expected to dominate the other by SSD.

In this study, ex-post data will be used to estimate the premium required by bondholders in the <u>past</u> for the probability of default in the <u>future</u>. As a result, the

conclusions will pertain to the probability of default which was assessed by the market in the past. Of course, if additional information regarding the future becomes available, the probability of default in the future may change. Thus, the assumption is not made that ex-post distributions are necessarily stable or that they represent ex-ante distributions. Rather, ex-post data are assumed to incorporate the market's assessment at a given point in time of the possibility of default in the future.

It is reasonable to assume that the probability of default as assessed by the market may vary from year to year. For practical reasons (and due to statistical limitations) default risk as attributed to each and every year will not be measured. Rather, the study distinguishes between years of economic recession and prosperity, with the expectation that the derived probability of default will be larger in years of economic recession or contraction than in years of economic prosperity or stability.

Finally, before moving to the derivation of and illustrations of the use of this technique, it should be noted that this analysis can logically be extended to the portfolio setting under quite general conditions (refer to Kroll, 1981, for the necessary framework). Obviously in practice the investor may diversify either bond with other assets rather than be limited to the assumption that he/she buys either the Aaa corporate bond or the government bond.

The Definition of Default

In a one-period setting with no taxes, the holder of a bond which matures at the end of that period is promised a return of (1 + C), where C is the coupon rate. Risk for the bondholder is the probability of realizing a return less than the promised return as a result of the firm defaulting on the bond agreement. Default occurs when the firm has generated earnings before interest and taxes (EBIT) less than the principal and interest legally owed to the bondholder at maturity. There exists a distribution of possible levels of EBIT, only one of which will be the outcome at the end of the period. Thus, there are two equivalent statements of the default risk inherent in the ownership of a bond:

$$\text{Default risk} = \Pr[(1+r) < (1+C)] \qquad (1)$$

where r is the realized rate of return on the bond and alternatively,

$$\text{Default risk} = \Pr[\text{EBIT} < (1 + C)B] \qquad (2)$$

where B is the face value of the bond.

The differences among bonds in the probability of default should be reflected in the market value of the bonds and hence in the distributions of returns (and distributions of EBIT) when bonds of different rating categories are compared.

Measuring the Probability of Default

Define two firms, F and G, which differ in their distributions of EBIT such that firm G has the greater probability of default. Let δ represent the greater probability of default inherent in firm G's EBIT distribution. If the bonds of F and of G are matched in all respects except for the probability of default, it is possible to derive that value of δ such that when it is incorporated in G's EBIT distribution, an investor (with a utility function of a given class of utility functions) will derive greater or equal expected utility from investing in F's bonds when compared with G's bonds. In the marginal case, the investor would be exactly indifferent between selecting F and selecting G as an investment.

The Stochastic Dominance Criteria when applied to two distributions insure that if F dominates G by the given degree of stochastic dominance (first, second or third degree), then all investors with a utility function which is a member of the associated set of utility functions (first degree, $u' \geq 0$; second degree, $u' \geq 0$, $u'' \leq 0$; third degree, $u' \geq 0$, $u'' \leq 0$, $u''' \geq 0$ will gain greater or equal expected utility from investing in F. The Stochastic Dominance Criteria are based on the von Neumann-Morgenstern axioms. If these axioms hold, and as a result F is preferred to G, then it follows that the expected utility of F is greater than the expected utility of G. Thus, we can state

the relationship between expected utility and stochastic dominance as follows:

$$E_F U(x) \geq E_G U(x) \iff F \text{ dominates } G \text{ (or FDG)}$$

As a result of this analysis, we justify applying stochastic dominance as a technique for quantifying the probability of default.

In deriving δ, we shall consider two cases:

Case 1: Default results in zero return.

Case 2: Default results in a return greater than zero but less than the promised return of $(1 + C)$.

The return to the bondholder in Case 1, $(1 + r)$, is as follows:

$$(1 + r) = \begin{cases} 0 & \text{if default} \\ (1 + C) & \text{if no default} \end{cases}$$

The return to the bondholder in Case 2, $(1 + r)B$, where EBIT is defined as Y, is as follows:

$$(1 + r)B = \begin{cases} (0) \int_{-\infty}^{0} Yf(Y)dY + \int_{0}^{(1+C)B} Yf(Y)dY & \text{if default} \\ (1 + C)B & \text{if no default} \end{cases}$$

As we have defined δ as the probability of default which when incorporated in the risky distribution would make the marginal risk-averse investor indifferent between F and G, we can make equivalent the expected outcome from investing

in F and from investing in G. For this derivation, we shall use Aaa bonds as representing the investment with default risk and government bonds (GOVT) as the default-free investment.

As this study considers only bonds which did not default we expect F_{Aaa} to dominate F_G and that F_{Aaa} will thus be preferred by all risk-averters (and maybe by all investors). As a result, we expect to find

$$\int_{-\infty}^{x} [F_G(t) - F_{Aaa}(t)]dt \geq 0$$

for all values of x. Such a result implies that all risk-averse investors would be better off by investing in F_{Aaa} rather than in F_G. However, this conclusion is incorrect, as corporate bonds are exposed to default risk whereas government bonds are not. Thus, we change F_{Aaa} by incorporating the market estimate of default until

$$\int_{-\infty}^{x} [F_G(t) - F'_{Aaa}(t)]dt$$

is negative for at least one value of x, that is, until the dominance exactly disappears (where F'_{Aaa} is the F_{Aaa} distribution revised into incorporate default risk). That value of δ which causes the dominance to disappear is the market estimate of the risk of default.

In many cases, we observe empirically that the two distributions F_G and F_{Aaa} intersect only once (or do not intersect at all), F_{Aaa} intersects F_G from below, and the mean return of F_{Aaa} is greater than the mean return of F_G. In this specific case, every risk-averse investor would prefer F_{Aaa}

over F_G if and only if $E_{Aaa}(x) \geq E_G(x)$. So, we change the distribution F_{Aaa} and hence the mean of F_{Aaa} until the dominance disappears.

To be more precise, in case I above, the expected return on the investment in Aaa bonds, once the probability of default is incorporated, is

$$\delta(0) + (1-\delta)E_{Aaa}(x)$$

where $E_{Aaa}(x)$ is the expected value of the distribution of rates of return on the Aaa bond. We set this equal to the expected return on the government bond $[E_{GOVT}(x)]$, then solve for δ:

$$\delta(0) + (1-\delta)E_{Aaa}(x) = E_{GOVT}(x)$$

$$\delta = \frac{E_{GOVT}(x)}{E_{Aaa}(x)} \qquad (3)$$

A graphical illustration of the Aaa' distribution which results from including δ is displayed in Figure 3. The inclusion of δ probability of default in the Aaa distribution has resulted in a shift in Aaa's cumulative probability distribution such that the revised Aaa distribution and the GOVT distribution intersect at δ. As a result, whereas initially Aaa D GOVT by FSD, the inclusion of δ has caused dominance to reverse so that GOVT D Aaa' by SSD. The cumulative difference in the areas under the two curves is exactly zero, and there is no value of x for which it is negative. Thus, GOVT exactly dominates Aaa' by SSD.

In Case 2, the expected value of an investment in Aaa bonds, once the probability of default is incorporated, is

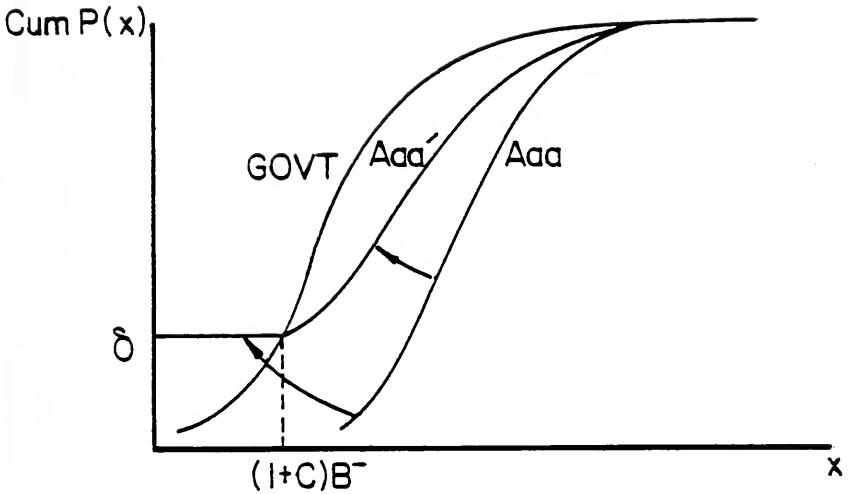


Figure 3. Shift in Aaa Distribution
(Case 1: Default Results in Zero Return)

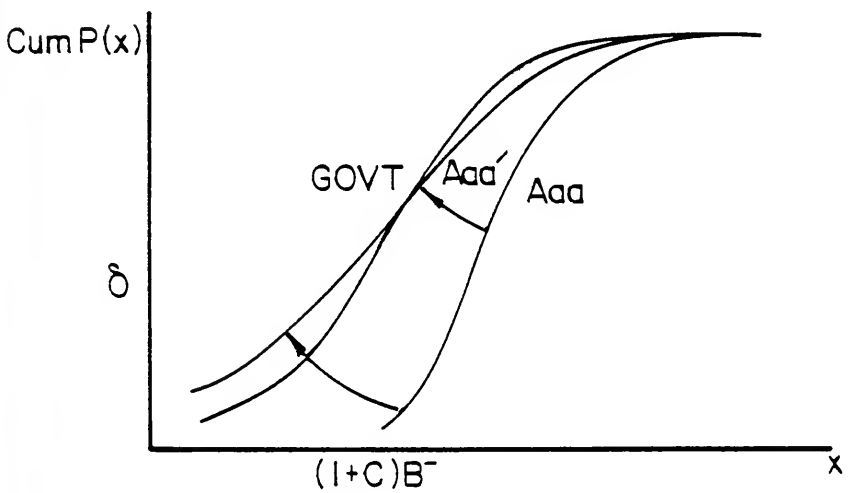


Figure 4. Shift in Aaa Distribution
(Case 2: Default Results in Return
Less Than Promised Return)

$$\delta[(0)\int_{-\infty}^{0} xf(x)dx + \int_{0}^{(1+C)B^{-}} xf(x)dx] + (1-\delta)E_{Aaa}(x)$$

We set this equal to the expected value of the government bond, then solve for δ.

$$\delta[(0)\int_{-\infty}^{0} xf(x)dx + \int_{0}^{(1+C)B^{-}} xf(x)dx] + (1-\delta)E_{Aaa}(x) = E_{GOVT}(x)$$

$$\delta = \frac{E_{GOVT}(x) - E_{Aaa}(x)}{(0)\int_{-\infty}^{0} xf(x)dx + \int_{0}^{(1+C)B^{-}} xf(x)dx - E_{Aaa}(x)} \qquad (4)$$

A graphical illustration of the shift in the Aaa distribution which results from including δ is displayed in Figure 4. In this case also, the inclusion of δ causes the Aaa cumulative probability distribution to shift so that it intersects with the GOVT distribution at δ. Again, the cumulative difference in the two distributions is zero, and there is no value of x for which the cumulative difference is negative. Whereas initially Aaa dominated GOVT by FSD, dominance has reversed by including δ so that GOVT exactly dominates Aaa' by SSD. The manner in which we derived δ assures that SSD dominance will result, that dominance will have reversed, and that the cumulative difference will be exactly zero. These results follow as a mathematical necessity from our solving for the precise value of δ that would, when included in the risky distribution, result in the expected outcomes being exactly identical.

To this point, the graphical analysis has been limited to those cases where there is no more than one point of

intersection between the two cumulative distributions. It is necessary to extend the work to include the derivation of δ in the general case of n-points of intersection. The derivation of an algorithm for the general case is presented below, and is based on Levy and Kroll's (1979) observation that in the case where the cumulative distributions are discrete it is only necessary to check the points of intersection when testing for stochastic dominance (Levy and Kroll, 1979, p. 126).

An Algorithm for Calculating the Probability of Default When There Are N Intersections Between the Cumulative Probability Distributions

For discrete distributions, as noted above, it is only necessary to check the points of intersection. This is the basis for developing the algorithm. For purpose of analysis, assume two discrete cumulative probability distributions of a given variable, which we call $F_{X'}(x)$ and $F_Y(y)$ where X' has default risk and Y does not, and proceed as follows:

1. Order (rank) the observations of the given variable for X' and then for Y from the smallest value to the largest.

2. As X' is defined as having some probability of default, we know that it contains some implied level of delta. Since we have discrete distributions, the cumulative probability at any point can be expressed as an interval as follows:

$$x' = \begin{cases} 0 & 0 < p \leq \delta \\ x_1' & \delta < p \leq [(1-\delta)(\frac{1}{n})+\delta] \\ x_2' & [(1-\delta)(\frac{1}{n})+\delta] < p \leq [2(1-\delta)(\frac{1}{n})+\delta] \\ x_3' & [2(1-\delta)(\frac{1}{n})+\delta] < p \leq [3(1-\delta)(\frac{1}{n})+\delta] \\ \vdots & \\ x_n' & [(n-1)(1-\delta)(\frac{1}{n})+\delta] < p \leq 1 \end{cases}$$

where n is the number of observations, and all observations are assigned the same probability $\frac{1}{n}$.

$$y = \begin{cases} y_1 & 0 < p \leq (\frac{1}{n}) \\ y_2 & (\frac{1}{n}) < p \leq (\frac{2}{n}) \\ y_3 & (\frac{2}{n}) < p \leq (\frac{3}{n}) \\ \vdots & \\ y_n & (\frac{n-1}{n}) < p \leq 1 \end{cases}$$

3. Calculate (y - x') for each change in probability. Call these areas, 1 . . . 2n. Formulas for each area (y - x') follow:

Area	y - x'
1.	$(y_1-0)\delta$
2.	$(y_1-x_1')(\frac{1}{n}-\delta)$
3.	$(y_2-x_1')[((\frac{1}{n})(1-\delta)+\delta)-(\frac{1}{n})]$
4.	$(y_2-x_2')[(\frac{2}{n})-((\frac{1}{n})(1-\delta)+\delta)]$
⋮	

2n-1. $(y_n - x'_{(n-1)})\left[\left(\frac{n-1}{n}\right)(1-\delta)+\delta\right] - \left(\frac{n-1}{n}\right)$

2n. $(y_n - x'_n)\left(\frac{n}{n} - \left[\left(\frac{n-1}{n}\right)(1-\delta)+\delta\right]\right)$

4. Begin with a large value for δ (so that YDX' by SSD) and reduce it until the cumulative difference is no longer greater than zero (i.e. until exact SSD results and any further reduction in δ would result in X'DY).

In order to present the algorithm, it is necessary to define the following variables:

$$\alpha_i = (y_{i+1} - x'_i)\left[\delta\left(1 - \frac{i}{n}\right)\right], \quad i=0, 1, \ldots, n-1 \qquad (5)$$

$$\text{and } \alpha_n = 0$$

$$\beta_i = (y_i - x'_i)\left[\frac{1}{n} + \delta\left(\frac{i-1}{n} - 1\right)\right], \quad i=1, \ldots, n \qquad (6)$$

$$\text{and } \beta_0 = 0$$

$$\psi = \alpha_i + \beta_i, \quad i = 0, 1, \ldots, n \qquad (7)$$

The following two rules must hold for the δ which results in precise SSD:

$$\text{Rule 1:} \quad \sum_{i=0}^{I} \psi_i \geq 0 \quad \text{for all } i \qquad (8)$$

$$\text{Rule 2:} \quad \sum_{i=1}^{I} \psi_i - \alpha_i \geq 0 \quad \text{for all } i \qquad (9)$$

The algorithm is based on the assumption that default results in a zero return to the bondholder (Case 1). As a result, the application of the algorithm to the data will yield market estimates of the probability of the worst possible outcome occurring. A comparison of the probability

of getting zero (δ for Case 1) with the probability of getting some outcome less than the promised outcome (δ for Case 2) yields the result that the latter δ is larger. A comparison of equation (3) with equation (4) yields the following observations:

(1) The numerators are identical.

(2) The denominator in equation (4) is smaller by the amount,

$$(0) \quad \int_{-\infty}^{0} xf(x)\,dx + \int_{0}^{(1+C)B^-} xf(x)\,dx$$

(3) As a result, the δ for Case 2 is larger than the δ for Case 1.

The economic interpretation of this result is straightforward. The probability of getting something, even though smaller than promised, is greater than the probability ot getting zero in the event the firm defaults. The mathematical implication is that a larger value of δ will be required to be incorporated in the distribution with default risk (Aaa in this example) so that it will precisely dominate the default-free option (GOVT in this example) by SSD. Because the algorithm is developed for Case 1, the resulting values of the relative probabilities of default when the empirical data are analyzed represent the minimum values of this measure as assessed by the market.

Numerical Examples

Example 1: (G is the default-free distribution, A is the distribution having default risk, A' is the A distribution after the inclusion of the implied δ.)

A		G		A'	
x	P(x)	x	P(x)	x	P(x)
1.08	0.25	1.08	0.25	0	δ
1.10	0.25	1.10	0.25	1.08	$(1-\delta)(0.25)$
1.18	0.25	1.12	0.25	1.10	$(1-\delta)(0.25)$
1.20	0.25	1.20	0.25	1.18	$(1-\delta)(0.25)$
$E_A(x) = 1.14$		$E_G(x) = 1.125$		1.20	$(1-\delta)(0.25)$

$$\delta = 1 - \frac{E_G(x)}{E_A(x)} = 0.01315789$$

We interpret this as meaning that the investment in option A exposes the investor to a 1.32% greater probability of default than would an investment in option G. Note that this formulation of δ calculates the probability of getting a zero return. The probability of getting a return less than the promised return, but greater than or equal to zero, of course, will be larger than 1.32%. This example yields a δ value in the range of those found in the empirical analysis. However, the magnitude of the delta of 1.32% prohibits a graphical illustration. 1.32% when graphed is so small that the resulting shift in the A distribution cannot readily be seen.

In order to graphically demonstrate the estimation of delta, the following example is one for which there is no economic definition of the two distributions. Rather, this

is a purely mathematical example. Define two alternatives which differ only according to a given attribute. Assume that alternative C possesses the attribute and that alternative D lacks it. We can then solve for the magnitude of this difference. For consistency, we shall again call this difference delta, but without defining delta as the relative probability of default. This example illustrates the more general use of this technique.

Example 2: (D possesses the given attribute, C does not possess it, and D' is the D distribution after the inclusion of the implied δ).

D	
x	P(x)
10	0.25
20	0.50
30	0.25
$E_D(x)=20$	

C	
x	P(x)
8	0.25
10	0.50
12	0.25
$E_C(x)=10$	

D'	
x	P(x)
0	δ
10	$(1-\delta)(0.25)$
20	$(1-\delta)(0.50)$
30	$(1-\delta)(0.25)$

$$\delta = 1 - \frac{E_C(x)}{E_D(x)} = 0.50$$

The selection of alternative D is accompanied by a 50% greater probability of incurring the given attribute than if alternative C had been chosen. Figure 5 provides a graphical demonstration of this example, and illustrates the shift in the D distribution when the implied delta is included.

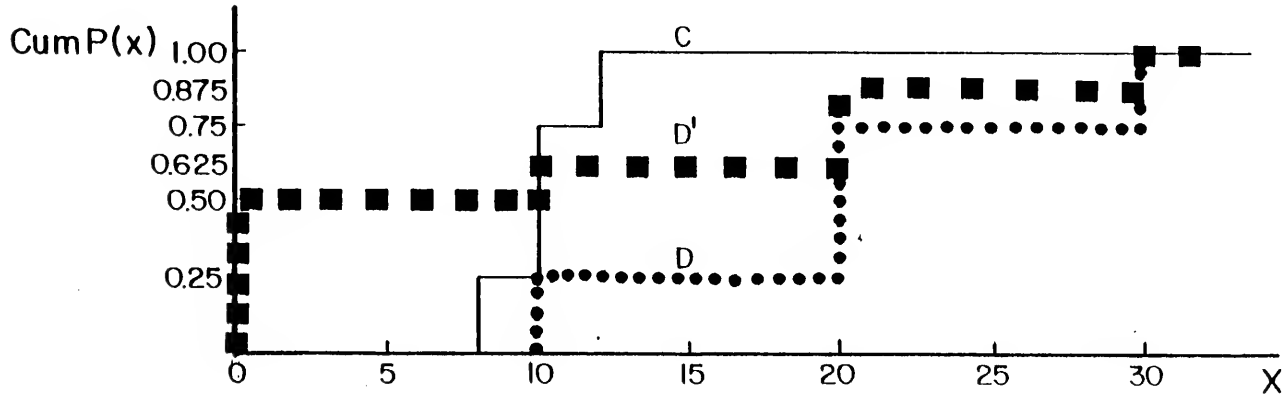


Figure 5. Cumulative Probability Distributions for Example 2 Alternatives C and D Demonstrating Shift in D to D' When Implied Delta is Included in D.

CHAPTER IV
METHODOLOGY AND RESULTS

Introduction

The significance of this application of stochastic dominance is in its potential for quantifying relationships. The technique developed in this paper has many potential applications, for it can be used to calculate the magnitude of differences in cumulative probability distributions. Thus, it can be used to quantify the magnitude of the remaining dimension along which distributions differ once all other relevant differences are neutralized.

The stochastic dominance technique developed in this paper is applied to data for groups of investment grade corporate bonds and government bonds which are comparable along all feasible dimensions except for bond rating. As the bond rating is assigned primarily on the basis of the bond rating agency's assessment of the relative probability of default prior to maturity, the bond rating serves as a proxy for the relative probability of default. The data are stratified according to bond rating category in order that the magnitude of the relative probabilities of default may be quantified by the stochastic dominance technique developed in this study. Market price data and corporate bond coupon

rates are used to calculate observations of holding period returns which are then analyzed for the relative probabilities of default faced by the investor with a short horizon (6 months in this study). Both real and nominal holding period return series are analyzed. For the investor with a longer horizon (in this study both 10 and 20 year data are analyzed) statistical limitations required the use of yield-to-maturity data rather than holding period return data. The yield-to-maturity series are relevant for the institutional investor who plans to hold the bond in one case for 10 years and in the other case for 20 years, for his/her investment horizon is then identical with the bond's term to maturity. When the investment horizon is not equal to the 10 or 20 years (or any other given term to maturity), it is necessary to analyze either holding period return series or yield-to-maturity series when the term to maturity is equal to the given investor's investment horizon. Failure to perform this matching of time to maturity with the length of the investor's horizon results in an incomplete neutralization of interest rate risk (type (a) risk in this model) and in a distortion of the relative probabilities of default calculated from such data. The likelihood of default by the firm increases in times of economic contraction and decreases in times of economic expansion. As a result, it is necessary to analyze the probability of default as a function of the state of the economy by considering both periods of economic contraction and of economic expansion.

Methodology

There are many characteristics of bonds which distinguish one from another. In a study of this nature, it is necessary to examine these distinguishing characteristics in order to identify those which contribute to the expected return on investment in them. The coupon rate reflects the magnitude of the promised cash flows over the remaining life of the bond, and the face value indicates the promised cash flow at maturity. Of course, when there is risk of default, the promised cash flows are not equal to the expected cash flows. Rather, in the case of bonds with default risk, the expected cash flows will be consistently less than the promised cash flows. The term to maturity indicates the length of time over which the investor is exposed to the risk of ownership (provided the bond is held to maturity). Callability is another source of risk to the bond holder. Its' impact on expected cash flows is the same as is the impact of default risk in that a call feature increases the probability that the actual return on the investment will be less than the promised return on the investment (the yield). Bonds differ also in sinking fund provisions (if any) and in status in the event of bankruptcy. The presence of a sinking fund and the presence of a collateral securing the debt (i.e. the bond) in the event of bankruptcy both serve to reduce the likelihood that the investor will experience a loss in the event the firm defaults. They do not reduce

the probability that the firm will default, but rather reduce the magnitude of the loss to the bondholder if default is not identical with bankruptcy. The legal definitions of the two conditions differ. Default exists when the firm is unable to meet a scheduled interest payment or is unable to repay the principal at maturity. At either such point, the firm is technically in default on the bond. The state of default exists when the firm is unable to meet the terms of the bond indenture. The state of bankruptcy exists when the firm has been judicially declared subject to having its' assets administered under the bankruptcy laws for the benefit of its creditors. Finally, bonds differ in the tax status of income to the investor. Interest income from corporate bonds and from U.S. government bonds is subject to federal income tax. However, interest income from U.S. government bonds is exempt from state and local tax levies (Reilly, 1982, p. 373). The treatment of capital gains income from U.S. government bonds by state and local taxing authorities varies. Ideally, one would analyze data on bonds matched for all relevant characteristics except bond rating in order to obtain accurate measures of the relative probabilities of default.

The goal, as stated in the introduction to the model (Section III), is to neutralize interest rate risk so as

to focus on default risk. We know that the duration[1] of a bond takes into account both the time over which the bond provides payments and the pattern of those payments over time, whereas the maturity of a bond considers only the first of these. As a result, it is suggested that duration measures on bond's characteristics more accurately (Sharpe, 1981, p. 88). For bondholders, interest rate risk consists of both price risk and coupon reinvestment risk. Bierwag and Kaufman (1977) have mathematically verified that both price risk and coupon reinvestment risk can be neutralized completely only when the duration of the bond equals the investor's holding period.[2] For these reasons, matching bonds by duration rather than by maturity is theoretically correct in order to neutralize interest rate risk. When bonds are subject to different degrees of default risk, however, duration is not an unambiguous measure of effective maturity because of the problem of expected cash flows being

---

[1]There are several definitions of duration, for duration is an index, and is simply the weighted average of the time periods at which the cash flows are received. The definitions of duration differ according to their assumptions about the stochastic process generating unexpected interest rate changes over the course of the investor's planning period.

[2]Kaufman (1980, p. 3) points out that immunization against interest rate risk is perfect only if the investor has selected the formulation of duration which is based on the stochastic process which actually generated the unexpected interest rate changes over the period the bond was held. This, of course, cannot be determined in advance.

less than promised cash flows (Weinstein, 1981, pp. 258-259). The calculation of duration uses the yield to maturity as the discount rate, and the promised coupon payments and maturity value as cash flows. It is possible for two bonds of differing degrees of default risk to have the same duration if they have the same term-to-maturity, the same coupon rate and the same yield-to-maturity. Duration measure D1 (Macaulay, 1938) is used to illustrate this point, where

$$D1 = \frac{\sum_{t=1}^{n} \frac{C_t(t)}{(1+YTM)^t}}{\sum_{t=1}^{n} \frac{C_t}{(1+YTM)^t}} \qquad (1)$$

where

t = the period in which the coupon and/or principal payment occurs

C_t = the interest and/or principal payment occurring in period t

YTM = the market yield-to-maturity on the bond

The formulation of duration measure D1 does not consider the possibility that the expected cash flows might be less than the promised cash flows (as is the case when the probability of default exists). If the expected cash flows are less than the promised cash flows, then the use of the risk-adjusted discount rate (the yield-to-maturity) is theoretically incorrect, as it results in the same type of distortion of D1 as was noted by Robichek and Myers (1966) in their

work on the calculation of net present value. As the calculation of duration is biased in the case where bonds are identical except for differences in default risk, we conclude that the matching of bonds of differing degrees of default risk for duration will not result in neutralizing interest rate risk. In addition, there are practical problems involved in identifying bond's durations in order to select them for inclusion in this study. As duration is not published along with the other items of publicly available bond data (as in Moody's, Standard and Poor's, and The Wall Street Journal's information), it would be necessary to calculate each bond's duration, then select those with the desired duration as a sample. Both the theoretical and practical problems identified led the researcher to decide against matching bonds for duration.

Description of Data and Presentation of Empirical Results

Data were collected and analyzed for investment grade corporate bonds and for government bonds. Both holding period return data and yield-to-maturity data were included in the study. The assumed investment horizons ranged from six months for the holding period return data to ten to twenty years for yield-to-maturity data. The sources of data were various issues of the Federal Reserve Bulletins for the yield-to-maturity data for a ten-year investment horizon and Moody's Bond Record for the six month holding

period return data assuming a six-month investment horizon and also for the yield-to-maturity data assuming a twenty year investment horizon. The influence of the level of economic activity on the magnitude of the relative probabilities of default was addressed by analyzing data for years of economic expansion and for years of economic contraction separately. The cyclical behavior of yield differentials between long-term Treasury bonds and Aaa and Baa corporate bonds is well documented, and is presented in many textbooks (Brigham, 1979, as an example). Historically, the yield differentials widen in periods of economic contraction and narrow in periods of economic expansion. In order to account for the impact of the level of economic activity on the magnitude of the relative probabilities of default, data were sorted into sub-sets consisting of periods of economic expansion and of economic contraction as identified and defined by the National Bureau of Economic Research, Inc. (1981, p. 21). Expansionary periods were identified as 1969, 1971 through 1973, and 1976 through 1979. Periods of economic contraction were identified as 1970, 1974 through 1975, and also 1980. Both nominal and real data were studied for the six-month holding period return analysis. Nominal data were converted to real data by using the percent change in the Consumer Price Index (CPI) unadjusted for seasonal differences, for all urban consumers for each six month period. The relationship between the nominal and the real holding period return is as follows:

$$(1 + R_N) = (1 + R_R)(1 + h) \tag{2}$$

where R_N is the nominal holding period return, R_R is the real holding period return, and h represents the realized rate of inflation for the period. The value of h used to convert nominal data to real data was the percent change in the CPI from the beginning to the end of the six month period. The total nominal six month holding period return was calculated as follows:

$$(1 + R_N) = \frac{P_1 + C/2}{P_0} \tag{3}$$

where P_0 and P_1 are the market price of the bond in dollars at the beginning and at the end of the period, and C is the annual coupon expressed in dollars.

The relative probabilities of default displayed in Tables 3 through 7 are the result of applying the algorithm derived in this paper (Chapter III) for the general case of n-points of intersection between cumulative probability distributions. The computer program written to do this reiteratively tests until it has found the last value of delta for which both rules of the algorithm hold. This value of delta is the smallest value of delta for which the cumulative difference in the cumulative probability distributions is greater than or equal to zero. A reduction in the value of delta by 0.001 would result in the cumulative difference being strictly less than zero. The values

of delta in the tables are to be read as percents, that is, 0.013 is read as 1.3 percent.

The first data analyzed consisted of nominal six month holding period returns. The sample consisted of 465 bonds matched for maturity (20 years) and for the presence of a sinking fund, for callability, and for the lack of subordination. The bonds were all investment grade corporate bonds with the above listed characteristics. The sample was stratified, where the strata were the four investment grade bond rating categories (Aaa, Aa, A, and Baa). The bonds for a given rating category were combined in portfolios of either 10 or the population, whichever was smaller. The source of the data was Moody's Bond Record. For each portfolio, the semi-annual holding period return was calculated. As the period covered was 1969 through 1980, there were 24 observations of semi-annual holding period returns for each bond rating category for the total period, 16 for periods of economic expansion and 8 for periods of economic contraction. Tables 3 and 4 present the relative probabilities of default calculated by the algorithm presented in this paper as it is applied to distributions of six-month nominal and six-month real holding period returns respectively. There were an insufficient number of comparable government bonds (20 years to maturity) to permit their inclusion. Appendix D presents the values for the six-month total holding period returns both nominal and real, which were

Table 3. Relative Probabilities of Default of Corporate Bonds When the Investment Horizon is Six Months
(Calculated using nominal data)
(Based on 1969-1980 Moody data)

	Aaa vs. Aa	Aaa vs. A	Aaa vs. Baa	Aa vs. A	Aa vs. Baa	A vs. Baa
Total Period	0.004	0.010	0.009	0.009	0.006	0.003
Economic Expansion	0.012	0.018	0.019	0.008	0.008	0.008
Economic Contraction	0.007	0.007	0.010	0.011	0.011	0.003

Table 4. Relative Probabilities of Default of Corporate Bonds When the Investment Horizon is Six Months
(Calculated using real data)
(Based on 1969-1980 Moody data)

	Aaa vs. Aa	Aaa vs. A	Aaa vs. Baa	Aa vs. A	Aa vs. Baa	A vs. Baa
Total Period	0.004	0.010	0.009	0.009	0.006	0.004
Economic Expansion	0.012	0.018	0.019	0.008	0.008	0.009
Economic Contraction	0.007	0.006	0.010	0.011	0.008	0.003

used to calculate the values of delta. Appendix E presents the values of (1+h) used to convert nominal data to real data.

Various issues of the Federal Reserve Bulletin were the source of yield-to-maturity data on Aaa and Baa corporate bonds and on government bonds. These data were used to calculate the relative probabilities of default of corporate bonds and of government bonds when the investment horizon is ten years or more. These relative probabilities of default are displayed in Table 5, both for the total period of 1971

Table 5. Relative Probabilities of Default of Corporate Bonds and Government Bonds When the Investment Horizon is Ten Years (Based on 1971-1980 Federal Reserve Data)

	Aaa vs. Govt.	Baa vs. Govt.	Baa vs. Aaa
Total Period	0.013	0.023	0.011
Economic Expansion	0.012	0.021	0.010
Economic Contraction	0.014	0.027	0.013

through 1980 and for sub-periods of economic expansion and of economic contraction. The yield-to-maturity data published in the Federal Reserve Bulletin for corporate bonds are based on averages of daily figures from Moody's Investor's Service, and are for seasoned issues. The government bond data as published by the Federal Reserve are yields-to-maturity for all government bonds neither due nor callable in less than ten years. The relative probabilities of default presented in Table 5 are calculated by the model

developed in this paper and are based on monthly observations of yields-to-maturity. As a result, there are 120 observations for the total period, 84 for periods of economic expansion and 36 for periods of economic contraction.

Tables 6 and 7 present relative probabilities of default for corporate bonds when the investment horizon is twenty years. These calculations are based on the same set of data from Moody's Bond Record which was used to calculate the relative probabilities of default when the investment horizon is six months (presented in Tables 3 and 4). Table 6 is based on data for 1971 through 1980, while Table 7 is based on data for 1969 through 1980. As a result, the number of observations for the total period are 20 for Table 6 and 24 for Table 7. For periods of economic expansion, the number of observations for Tables 6 and 7 are 14 and 16 respectively, and for periods of economic contraction, the numbers of observations are 6 and 8 also respectively.

Analysis of Results

There are two aspects to analyzing empirical results, one of which is peculiar to research projects in economics, and the other of which is applicable to all empirical research. As this study addresses an economic question, i.e., what is the market assessment of the relative probabilities of default, it is necessary to provide an economic interpretation of the results presented in the study. In

Table 6. Relative Probabilities of Default of Corporate Bonds When the Investment Horizon is Twenty Years (Based on 1971-1980 Moody Data)

	Aa vs. Aaa	A vs. Aaa	Baa vs. Aaa	A vs. Aa	Baa vs. Aa	Baa vs. A
Total Period	0.003	0.006	0.012	0.003	0.010	0.007
Economic Expansion	0.003	0.005	0.011	0.003	0.009	0.006
Economic Contraction	0.004	0.007	0.014	0.003	0.012	0.009

Table 7. Relative Probabilities of Default of Corporate Bonds When the Investment Horizon is Twenty Years (Based on 1969-1980 Moody Data)

	Aa vs. Aaa	A vs. Aaa	Baa vs. Aaa	A vs. Aa	Baa vs. Aa	Baa vs. A
Total Period	0.003	0.005	0.011	0.003	0.009	0.007
Economic Expansion	0.002	0.005	0.010	0.003	0.008	0.006
Economic Contraction	0.003	0.006	0.014	0.003	0.011	0.008

order to do this, three aspects of the economic interpretation of relative probabilities of default are analyzed: (1) the additivity characteristics of deltas; (2) the relationship of delta to the level of economic activity, including both the relative magnitudes of deltas and the relative sensitivities of deltas to a change in the level of economic activity; and (3) the relationship of delta to the length of the investment horizon. Secondly, it is necessary to examine the validity of the techniques employed in the research, for the results are only as valid as are the techniques employed in obtaining them. In addressing this question, the sensitivity of the results to the data grouping technique employed is tested. In addition, it is necessary to compare the relative probabilities of default generated in this study with the relative incidences of default reported in the literature. As a result, both the internal and the external validity of the research will be examined. As the deltas presented in this study are relative probabilities of default, they should be amenable to being represented graphically as are risk premia. When historical risk premia are plotted, one observes that the relative magnitudes of the risk premia are consistently positively correlated with the quality of the bond as indicated by the bond rating. The premia of Aaa corporate bonds over Treasury (government) bonds, of Baa corporate bonds over Aaa corporate bonds, and of Baa over government bonds are consistently positive.

In addition, when periods of economic contraction occur, the magnitudes of these relative risk premia increase. There are, of course, economic rationales for these observations which should also apply to the results of this study. The relative probability of default of high grade corporate bonds (say Aaa) over the government bonds is expected to be less than the relative probability of default of low grade corporate bonds (say Baa) over the same government bonds, as the quality of the bond as indicated by the rating is inversely related to the probability of default. The relative riskiness of bonds is reflected in historical differences in risk premia. It is necessary to determine if relative probabilities of default are additive, as are risk premia. In order to do this, it is first necessary to assume that bond rating assignments correctly rank bonds according to their relative probabilities of default. If one does not make this assumption, then the question is actually a joint hypothesis: (1) assigned bond ratings indeed rank bonds accurately by relative probability of default prior to maturity; and (2) the technique developed in this paper actually calculates relative probabilities of default which are economically valid. The literature, reviewed above, suggests that bond ratings do correctly rank bonds according to their relative probabilities of default as indicated by the historic incidence of default. It is necessary to determine the mathematical relationship between relative

probabilities of default, and then to test the deltas presented in Tables 3-6 for this relationship.

Consider three investment alternatives, B, A, and G and interpret them such that B is more likely to default than is A, and A is more likely to default than is G. Assume that the only dimension along which they differ is the probability of default. Define δ_1 as the relative probability of default of B over G, δ_2 as the relative probability of default of A over G, and δ_3 as the relative probability of default of B over A. If the deltas are strictly additive, then,

$$\delta_1 = \delta_2 + \delta_3 \tag{4}$$

We know that in the simple case of no more than one intersection between the cumulative probability distributions and when default results in a return of zero,

$$(1 - \delta_1)E_B(x) = E_G(x) \tag{5}$$

$$(1 - \delta_2)E_A(x) = E_G(x) \tag{6}$$

$$(1 - \delta_3)E_B(x) = E_A(x) \tag{7}$$

Subtracting (6) from (5) yields,

$$(1 - \delta_1)E_B(x) - (1 - \delta_2)E_A(x) = 0$$

Rearrange terms, so that,

$$(1 - \delta_1)E_B(x) = (1 - \delta_2)E_A(x)$$

Substitute (7) for $E_A(x)$, so that

$$(1 - \delta_1)E_B(x) = (1 - \delta_2)(1 - \delta_3)E_B(x)$$

Divide both sides by $E_B(x)$, so that

$$(1 - \delta_1) = (1 - \delta_2)(1 - \delta_3)$$

Expand right hand side

$$(1 - \delta_1) = (1 - \delta_2 - \delta_3 + \delta_2\delta_3)$$

Multiply both sides by (-1) and solve for δ_1

$$\delta_1 = \delta_2 + \delta_3 - \delta_2\delta_3 \qquad (8)$$

So, in this case, the deltas should be additive, but less a cross product term. Note that we assume that G was default-risk free. Equation 8 is applied to the data in Table 5 to produce Table 8 which demonstrates the nature of the additivity of deltas. An examination of Table 8 leads to the conclusion that relative default probabilities are additive, as the differences in the calculated total range of delta and the observed range of delta are virtually zero. As the relationships were derived for a pairwise comparison in which not all members possessed default risk, it was not correct to demonstrate additivity for any other sets of deltas. Thus, it seems that the data confirm the hypothesis that the relative probabilities of default as calculated by this technique are additive as are risk premia. The mathematical relationship existing within the structure of deltas

Table 8. Additivity of Relative Probabilities of Default as Demonstrated by Table 5 Data

	Baa vs. Aaa (1)	Aaa vs. Govt. (2)	Value of (1)+(2)-(1)(2) (3)	Estimated Baa vs. Govt. (4)	Difference (3)-(4)
Total	0.011	0.013	0.022857	0.023	(0.000143)
Economic Expansion	0.010	0.012	0.021880	0.021	0.000880
Economic Contraction	0.013	0.014	0.026182	0.027	(0.000818)

has been determined to exist in deltas which are calculated from empirical data. Thus, there exists a default risk structure composed of relative probabilities of default. In this paper, we have derived this structure for three different investment horizons, for the default risk structure relevant for a given individual is a function of his/her investment horizon. The default risk structure for a given investment horizon, of course, is based on the market's assessment of the relative probabilities of default for the different bond rating categories.

The default risk structure is also a function of the level of economic activity. As the probability of default is related to the state of the economy, the magnitudes of the relative probabilities of default are expected to be greater in periods of economic contraction than in periods of economic expansion. An examination of Tables 5 through 7 leads to the conclusion that the relative probabilities of default for every pairwise comparison except one (A vs. Aa is unchanged) are larger in periods of economic contraction than they are in periods of economic expansion. The fact that the A vs. Aa comparison is unchanged is not significant, as this comparison is not present in Table 5, and Tables 6 and 7 are based essentially on the same data (only the number of years covered varies). Essentially, then this A vs. Aa observation, as the only one not as predicted, represents one calculation out of nine (three pairwise comparisons in

Table 5, and six in Tables 6 and 7 together). These results are exactly as we would hope they would be if indeed the stochastic dominance measure of the relative probability of default has economic validity.

Before moving to an analysis of the impact of the length of the investment horizon on the magnitudes of the relative probabilities of default, it is necessary to assign a more precise interpretation to the values of delta as presented in Tables 5 and 6. There are two distinctive features of the data used to calculate Table 5 deltas which need elaboration and which affect the comparison of these results with those of Table 6. First, as was noted earlier, Table 5 data are for government bonds which are neither due nor callable in less than ten years and for corporate bonds with ten years to maturity whereas the data for Table 6 are for callable bonds with twenty years to maturity. Thus, the ten-year (or more) horizon deltas and the twenty-year horizon deltas differ in the degree which maturity has been held constant. As a result, the Table 5 deltas are biased when compared with Table 6 deltas, but there is no reason to assume bias <u>within</u> the Table 5 results when corporate bonds are compared with one another. As corporate bonds in general possess a call feature, no bias is expected to exist either in Table 5 Baa versus Aaa deltas or in any pairwise comparisons in Table 6 (or Table 7). Rather, a call feature is assumed for all corporate bonds with no difference between bond rating

categories as to its presence or lack of presence. There is a difference in callability as noted when government bonds and corporate bonds are compared, as in Table 5. The difference in the degree to which maturity is held constant in the 10-year versus the 20-year data and the difference in callability of government bonds and corporate bonds impinge on the strict interpretation of the calculated relative probabilities of default for the 10-year investment horizon when compared with the 20-year investment horizon.

As the corporate bond data for deltas presented in Table 5 and in Table 6 are characterized by maturity matching of 10 and of 20 year respectively, and by no bias expected in the treatment of callability, it is valid to analyze deltas calculated for the pairwise comparison of Baa corporate bonds versus Aaa corporate bonds. Such a comparison is presented in Table 9. It should be noted that the relative probability of default consistently increases with the length of the investment horizon. The relative probability of default of Baa over Aaa corporate bonds is consistently greater for the 20 year investment horizon than for the 10 year investment horizon. It should be noted that the magnitude of the difference is quite small, but the fact that the value of (b)-(a) is consistently 0.003 and the value of (1)-(2) is consistently 0.001 lends credibility to the suggestion that these results are not a mere statistical fluke. This conclusion holds for the total period covered by the data

Table 9. The Impact of the Investment Horizon on the Magnitude of the Relative Probability of Default of Baa Corporate Bonds Over Aaa Corporate Bonds

	20 Year Horizon Baa vs. Aaa (1)	10 Year Horizon Baa vs. Aaa (2)	Difference in Delta (1)-(2)
Total Period	0.012	0.011	0.001
Economic Expansion (a)	0.011	0.010	0.001
Economic Contraction (b)	0.014	0.013	0.001
Difference in Delta (b)-(a)	0.003	0.003	0.000

(1971 through 1980) as well as for sub-periods of economic expansion and of economic contraction. It is interesting to note that the differences in the relative probabilities of default (reflected in column 3) are consistently of the same magnitude, and do not vary with the definition of the period (total, economic expansion, or economic contraction). It also should be noted that the difference in the magnitude of delta when periods of economic expansion are compared with periods of economic contraction is invariant to the length of the investment horizon. For the 10-year investment horizon, the difference in the sub-period's deltas is 0.003 and for the 20-year investment horizon, the difference in the sub-period's deltas is the same 0.003. Thus, one can conclude from this that the increases in deltas when the 20-year investment horizon is compared with the 10-year investment

horizon are a function of the difference in the horizon. In this case, the difference in the relative probability of default is not affected by the state of economic activity, but rather only by the length of the investment horizon. In other words, the level of economic activity has no more effect on the default risk faced by an investor with a 20-year investment horizon than it does on an investor with a 10-year investment horizon. The difference in the length of the investment horizon is the crucial factor in the amount of default risk assumed by investment in a given rating category of corporate bonds. This issue is explored further in Chapter V in the discussion of future research.

Before moving to addressing the validity of this research, it is necessary to examine the relative probabilities of default when the investment horizon is short (six months in this study). These data are presented in Tables 3 and 4, but have not been analyzed to this point. The six month holding period return data which are the basis for these tables are nominal returns in Table 3 and real returns in Table 4 as noted earlier when these tables were initially presented. The results from using nominal versus real data are not significantly different from one another. Neither presentation of the relative probabilities of default when the holding period (the investment horizon) is six months has the economic validity observed in comparable presentations for the 10-year and 20-year investment horizons. With two

exceptions (A vs. Aa increases and Baa vs. A is unchanged) the magnitudes of the relative probabilities of default are smaller in periods of economic contraction than they are in periods of economic expansion. We expect the reverse to be true, as was observed in the 10-year and in the 20-year investment horizon series. In addition, the relationships predicted by the concept of additivity hold only for the sub-periods of economic expansion. It should be noted that the data set used for calculating six-month holding period returns (Tables 3 and 4) is the same data set which is the basis for the 20-year investment horizon analysis (Table 7). They are based on the same set of bonds. The only difference is that observations of market prices and coupon rates were used for the six-month holding period return calculations (Tables 3 and 4), and observations of yield-to-maturity were used for the 20-year investment horizon calculations (Table 7 is for the same period of 1969 through 1980, and Table 6 is for a sub-set of 1971 through 1980). The explanation for the inconsistent results does not seem to lie in the data themselves, but rather in either the choice of the horizon (6 months) or the use of holding period return data rather than yield-to-maturity data to calculate relative probabilities of default. As stated earlier, bond ratings are assigned on the basis of the bond rating agency's assessment of the relative probability that the bond issuer will default on the bond prior to maturity. We can interpret this as

meaning that the relative probabilities of default as reflected in the market's assessment of default probability (in market data) are based on the probability of default occurring prior to maturity. This is evidence that the relative probabilities of default as calculated by the technique developed in this study are to be interpreted as the probability of default before maturity. The market seems to be assessing the relative likelihoods of default prior to maturity. This interpretation lends support to the use of yield-to-maturity series where the time to maturity is equal to the investor's investment horizon as a basis for calculating delta. This is a reasonable conclusion if the market is indeed interpreting bond ratings as ranking bonds according to the probability of default prior to maturity, as rating agencies intend.

Although the results of the analysis of the yield-to-maturity series have been demonstrated to have economic validity, the possibility remains that the results were biased due to the theoretical and practical reasons leading to the preclusion of duration-matched data. The impact of duration on the calculation of delta is examined by analyzing the sensitivity of delta to duration measure D_1 and by testing the impact of the portfolio grouping of data on the sensitivity of delta to duration.

Table 10 analyzes the 20-year investment horizon yield-to-maturity data for sensitivity of the value of delta to a

Table 10. Analyze Yield-To-Maturity Data (Moody, 1969-1980)(Maturity=20) For Sensitivity of Delta to Change in D_1 (Macaulay's Measure of Duration)

	Aaa vs. Aa	Aaa vs. A	Aaa vs. Baa	Aa vs. A	Aa vs. Baa	A vs. Baa
(a) Range of D_1:	10.00000 to 10.99999					
Total Period (n=28)	- - -*	- - -*	0.002	- - -*	0.002	0.003
Economic Expansion (n=20)	0.002	- - -*	0.003	- - -*	- - -*	0.003
Economic Contraction (n=6)	- - -*	- - -*	0.004	0.005	0.007	0.003
(b) Range of D_1:	9.50000 to 11.49999					
Total Period (n=51)	0.002	- - -*	0.006	- - -*	0.005	0.007
Economic Expansion (n=37)	0.003	- - -*	0.005	- - -*	0.003	0.005
Economic Contraction (n=12)	- - -*	- - -*	0.003	0.002	0.007	0.006

Table 10 continued

	Aaa vs. Aa	Aaa vs. A	Aaa vs. Baa	Aa vs. A	Aa vs. Baa	A vs. Baa
(c) Range of D_1:	9.00000 to 11.99999					
Total Period (n=76)	0.002	- - -*	0.007	- - -*	0.006	0.007
Economic Expansion (n=51)	0.003	0.002	0.008	- - -*	0.006	0.007
Economic Contraction (n=25)	- - -*	- - -*	0.004	0.002	0.007	0.006
(d) Range of D_1:	8.50000 to 12.49999					
Total Period (n=99)	0.002	0.002	0.009	- - -*	0.008	0.007
Economic Expansion (n=66)	0.002	0.002	0.011	- - -*	0.009	0.010
Economic Contraction (n=27)	- - -*	- - -*	0.005	0.002	0.006	0.005

*For these pairwise comparisons, there exists no positive value of delta such that the cumulative difference is greater than or equal to zero.

change in the range of duration measure D_1. In order to produce Table 10, the range of D_1 was increased from a range of (10.00 to 10.99) to a range of (8.50 to 12.49) by increments of 1.00.

Table 10 indicates that delta has some sensitivity to a change in duration measure D_1. It is possible that the grouping procedure used to prepare the bond data (i.e., bonds combined in portfolios, then delta calculated for portfolio data) introduced a bias which is mathematical in nature. In order to examine the extent to which the calculations of delta based on data combined into portfolios was influenced (biased) by differences in the magnitude of D_1, Table 11 presents moments of the distributions of D_1 calculated both

Table 11. Analysis of the Variable D_1 for Data Used in Calculating Delta as Presented in Table 7 and in Table 10

		n	$\bar{x}$	σ^2
(a)	465 Bonds Individually			
	Aaa	114	11.3370	1.2951
	Aa	120	10.7913	1.3925
	A	119	10.3954	1.6455
	Baa	112	10.0333	1.1982
	Total Bonds	465		
(b)	465 Bonds in Portfolios			
	Aaa	12	11.2018	0.9811
	Aa	12	10.7238	1.1480
	A	12	10.4583	1.2098
	Baa	12	10.2808	1.6595
	Total Bonds	48		

for individual bonds (as presented in Table 10) and for portfolios of bonds (as presented in Table 7). If the data in Table 11a are not significantly different from the data in 11b, then the relative probabilities of default are not significantly biased by the methodology which grouped the bond data into portfolios. It was first necessary to determine if the values of D_i underlying (a) and (b) can be assumed to be a random sample from a normal distribution. The SAS computer software package was used to address this question. Table 12 presents the results. For Table 11(a), the modified Kolmogorov-Smirnov D-statistic was calculated and tested with the Null hypothesis being that the data are a random sample drawn from a normal distribution. For Table 11(b), the Shapiro-Wilk W-statistic was calculated and tested with the Null hypothesis again being that the data are a random sample drawn from a normal distribution. The statistics indicate that the null hypothesis cannot be rejected. Finally, it is necessary to see if the mean is the same and the variance is the same within each rating category. A Z-statistic was calculated for each rating category to see if the means are not statistically different from one another, and a F-statistic was calculated for each rating category to see if the variances are not statistically different from one another. These values are presented in Table 13. Table 13 results taken with Table 12 results indicate that the grouping of bonds into portfolios has not introduced a

Table 12. Statistical Analysis of the Distributions of D_1 in Order to Determine Whether Table 11(a) is Not Significantly Different From Table 11(b)

Rating	D	Prob>D	t(mean=0)	Prob>|t|	n
(a) Analysis of Table 11(a) (modified Kolmogorov-Smirnov D-statistic)					
Aaa	0.07380	0.129	106.4630	0.0001	114
Aa	0.12419	<0.010	100.1750	0.0001	120
A	0.10402	<0.010	88.4034	0.0001	119
Baa	0.06580	>0.150	97.0042	0.0001	112
(b) Analysis of Table 11(b) (Shapiro-Wilk W-statistic)					
Aaa	0.98140	0.964	39.1768	0.0001	12
Aa	0.95470	0.656	34.6720	0.0001	12
A	0.95910	0.714	32.9386	0.0001	12
Baa	0.97650	0.929	27.6456	0.0001	12

Table 13. Values of Statistical Tests for Equality of Means and of Variances Within Bond Rating Categories when D_1 is Calculated for Bonds Grouped in Portfolios and for Individual Bonds

Hypothesis	Test Statistic	Aaa	Aa	A	Baa
$H_0: \bar{x}_{(a)} = \bar{x}_{(b)}$	Z	0.443	0.206	(0.186)	(0.641)
$H_0: \sigma^2_{(a)} = \sigma^2_{(b)}$	F	1.320	1.213	1.360	0.722

significant bias due to the impact of duration (D_1) on the values of the relative probabilities of default. If anything, the fact that the variance of D_1 was reduced for each bond rating category when bonds are combined into portfolios would serve to reduce what impact not matching for duration might have had on the results.

Finally, although historical data on default experience is presented (Tables 1 and 2) one cannot compare these probabilities of default with the ones in this paper. The main factor which prevents such a comparison is that Table 1 and Table 2 data are not controlled for maturity, but rather are for all bonds of a given rating regardless of maturity. Thus, although the incidence of default can be interpreted as the probability of default over a given time period, Hickman's and Pye's data do not consider the investment horizon, and cannot be interpreted as the relevant probability of default for any given investor. In addition, of course, Hickman's and Pye's data cover a different time period than does this study.

CHAPTER V
SUMMARY AND CONCLUSIONS

Introduction

Bond rating agencies classify corporate bonds and state and municipal bonds into different categories according to the agencies' assessment that the issuer will default on the bond prior to maturity. The rank (bond rating category) assigned to a given bond is based primarily on publicly available information taken from the issuer's financial statements. Bond ratings rank bonds according to their relative probabilities of default as assessed by the bond rating agencies. Once one assumes that the bond market is efficient and is in equilibrium, then it follows that the market price of any given bond is a function of the risk incurred by ownership of that bond. There are two types of risk inherent in bonds: (a) interest rate risk (i.e., variability risk) and (b) default risk.

In this paper, a technique is proposed for quantifying default risk (type (b) risk). As a result, the probability of default as estimated by the bond market can be measured. The technique developed involves comparing the cumulative probability distributions of rates of return (or of yields-to-maturity) of bonds which differ only in default risk

once we have neutralized interest rate risk. A comparison of the cumulative probability distributions of say Aaa bonds with government bonds leads to the observation that the Aaa bonds dominate the government bonds by First Degree or by Second Degree Stochastic Dominance. In equilibrium, this observation cannot hold, for the Aaa bonds possess default risk and the government bonds do not. It follows then that the magnitude of the probability of default of the Aaa bonds over the government bonds can be quantified. The technique for quantifying this probability of default involves changing the distribution of rates of return (or yields-to-maturity) of the Aaa bond in a systematic manner until the dominance disappears. This manner of revising the Aaa distribution in order to include the implied probability of default results in the quantifying of the probability of default as assessed by the bond market.

Summary of Results

The significance of the technique developed in this paper is in its potential as a tool for quantifying relationships. It can be used to quantity the remaining dimension along which distributions differ after all other relevant differences are neutralized.

An algorithm for the stochastic dominance technique developed in this paper is applied to data for groups of

investment grade corporate bonds and government bonds which are comparable to the extent empirically feasible except for bond rating assignment. The bond rating serves as a proxy for the relative probability of default. Market price data and corporate bond coupon rates are used to calculate holding period returns which are then analyzed for the relative probabilities of default faced by an investor with a six-month investment horizon. Both real and nominal holding period return series were analyzed. Statistical limitations required the use of yield-to-maturity data rather than holding period return data for the investor with a longer horizon (both 10 and 20 year data were analyzed). In addition, as the magnitude of the probability of default is also a function of the state of the economy, it was necessary to calculate relative probabilities of default for both periods of economic contraction and of economic expansion.

Data were collected and analyzed for investment grade corporate bonds and for government bonds for a variety of investment horizons. Both holding period return data and yield-to-maturity data were studied. The sources of data were various issues of the Federal Reserve Bulletin and Moody's Bond Record. Definitions of periods of economic expansion and of economic contraction were provided by the National Bureau of Economic Research, Inc. (1981, p. 21). Nominal data were converted to real data by using the percent change in the Consumer Price Index.

The probabilities of default as calculated by the technique proposed in this paper were consistently in the direction predicted by economic theory. The magnitude of the probability of default was found to vary inversely with the quality of the bond as indicated by the bond rating. Lower grade bonds consistently exhibited a greater probability of default than did higher grade bonds. Also, the magnitude or default probability for a given bond rating category consistently was larger in periods of economic contraction than in periods of economic expansion. This result is as expected, since the likelihood of default for a given firm is expected to be related to the state of the economy. Finally, the probability of default was found to be positively related to the length of the investment horizon. It was consistently observed that the magnitude of the probability of default increased with the length of the investment horizon. This result held for all categories of bonds.

This calculation of the analysis of probabilities of default as assessed by the bond market is only one application of the technique developed in this paper. Other applications and extensions are discussed later in this chapter. The application of this technique to the quantifying of bond default probability, as assessed by the market, is characterized by limitations, as is the case when any such technique is applied in research. It is necessary to address limitations in the research methodology in order that the

analysis may be completed. The discussion of potential shortcomings of this research will be followed by suggestions for future research which is proposed in order to address these perceived limitations. It is suggested that the acknowledgment of research limitations followed by proposals for extensions of this present study in order to address these limitations should serve to bound this research without suggesting that it is complete in itself.

As the suggested major contribution of this paper is in its presentation of a technique for quantifying differences in cumulative probability distributions, the fact that the algorithm derived herein is for the case when default results in zero rather than in the more realistic result of a return less than expected, is a potential limitation. This is addressed by acknowledging that the empirical results must be interpreted as the minimum value of the implied probability of default. The results in this paper are interpreted as the market estimate of the likelihood of the worst possible outcome, and thus represent a lower bound on the estimated probability of default. In other words, the probability of getting zero in the event of default is less than the probability of default resulting in a return less than the expected return. The outcome in the event of default is represented by a continuum stated mathematically as,

$$\int_0^{(1+C)B} xf(x)\,dx$$

It is possible to extend this research to estimate empirically the magnitude of the implied probability of default when default results in the return as defined above. The derivation of the value of the expression for the magnitude of default probability in this case is presented in Equation (4) on page 42 of this study. It remains to test this equation empirically.

A second potential limitation of this research is that there is no statistical analysis of the results in order to determine whether the resulting relative probabilities of default are significantly different from one another. As the analysis of bond data was performed in order to demonstrate the use of the technique developed and presented in this study, rather than the focus of the paper being on the probabilities of default themselves, it is sufficient that the results are as economic theory would predict. Future research is proposed to analyze the measure presented in this paper as an alternative to bond ratings. This suggested extension actually consists of a series of related topics. When addressing these related topics, it is appropriate to consider the question of whether the results are significantly different from one another, or indeed whether they are significantly different from zero. Further elaboration on this is to be found in the next section of this chapter, where brief proposals for future research are outlined.

Another possible limitation of the research is seen in the use of data on portfolios of bonds rather than on individual bonds. The magnitudes of probabilities of default as calculated on portfolio data may not be relevant for individual bond selection. A degree of bias may be introduced by using portfolio averages of yields-to-maturity as the basis for calculating the relative probability of default. It is possible to address this question by calculating the probability of default for individual bonds of a given category, say Aaa, then determining if the average of these calculations is significantly different from the relative probability of default of Aaa bonds in general. The addressing of this question would fall logically under the suggested broad area of research on this measure of the relative probability of default as an alternative to bond ratings.

In addition, it may be that this research suffers from the limitation of being based on Moody's bond rating assignments only. This may be perceived as a narrowness of scope. As noted earlier, however, the suggested contribution of this paper is in the development and presentation of a technique which can be used to address many questions which are economic in nature. The selection of this particular question as a means to demonstrate the use of this technique is secondary to the development of the technique itself. The comparison of the probabilities of default for data stratified

by Moody's bond rating as compared with data stratified by rating assigned by an alternative bond rating agency (Standard and Poor, or maybe Fitch) remains to be done, and would follow logically either in the proposed study of bond market efficiency or in the proposed testing of this measure as an alternative to bond ratings.

A final suggested limitation of this research is in the selection of maturity and coupon rather than of duration for matching data. The theoretical justification for this decision is discussed in detail in Chapter IV, and is best summarized by the statement that duration is not always an unambiguous measure of effective maturity for bonds which differ in default risk. In addition, there exists a major practical barrier to studying large numbers of bonds matched for duration, as duration is not published, but rather must be calculated. Both theoretical reasons and practical reasons led the researcher to decide against matching bonds for duration. In addition, the impact of this decision on the results was statistically analyzed with the conclusion that no significant bias was found to have been introduced.

The discussion of possible limitations to the research leads logically into a discussion of proposals for future research. The following section provides brief outlines of further theoretical extensions of this technique and of other applications of this research.

Proposals for Future Research

Portfolio Implications Research

This study analyzed the risk of default incurred by holding a given bond in isolation. Of course, in reality bonds may be held in portfolios. They may be diversified with each other or they may be diversified with the market portfolio.

If one considers years (or periods) as states, we can apply State Contingent Stochastic Dominance (Kroll, 1981) as a technique for quantifying the probability of default when bonds can be diversified with each other or with other assets. The dominance by Second Degree State Contingent Stochastic Dominance ($S\hat{S}D$) of one option over another in the pairwise comparison guarantees the existence of dominance in the portfolio context (Kroll, 1981, pp. 10-11). In order to develop an $S\hat{S}D$ measure of default probability for bonds held in portfolios, it is crucial to justify viewing years (or periods) as states of the world. It is an empirical fact that bond returns (or yields) move together. When the return (or yield) on Aaa bonds has increased, one expects to observe that the return on say Baa bonds has also increased. The fact that bond returns move together suggests that years (or periods) may be treated as states of the world so that an $S\hat{S}D$ measure of the probability of default may be calculated for the bond data used in this study as well as for other

data. Future research to develop and test an SŜD measure of default risk is suggested as the mathematical and empirical work necessary is very similar to that presented in this paper. Such research is a logical extension of this paper and should yield results with quite general applications and interesting implications for portfolio theory.

In addition, the inclusion of the probability of default (δ) with a corresponding return of zero in the probability distribution of returns (or yields) for any bond with default risk permits the derivation of an expression comparable to the Capital Asset Pricing Model, but which includes a second constraint for the portfolio probability of default. Such an expression would represent the relevant probability of default when one can diversify with the riskless asset, and would represent the trade-off among risk as expressed by variance (σ^2), by default risk (δ) and the expected return (μ). The mathematics would involve minimizing the variance of the portfolio (σ_p^2) subject to two constraints:

$$(1) \quad \mu_p = \sum_{i=1}^{n} x_i \mu_i + (1 - \sum_{i=1}^{n} x_i) R_F$$

and $$(2) \quad \delta_p = \sum_{i=1}^{n} x_i \delta_i$$

where n represents the number of risky assets (i = 1, . . ., n), μ_i represents the expected return on the ith asset, and R_F represents the riskless asset. Alternatively, an expression could be derived by minimizing δ_p subject to a constraint

for μ_p. Such an expression would be relevant for and be of interest to a regulated industry such as banks. Both the derivation of an alternative to the CAPM and of a comparable expression for regulated industries and the empirical testing and analysis of such expressions is proposed for future research.

Finally, a measure for the probability of default can be derived when diversification with the risk-free asset is permitted by using the extension of stochastic dominance to this case, Stochastic Dominance with a Risk-Free Asset (SDR), as developed by Levy and Kroll (1978, 1979). Such a measure then could be applied to these data (or other sets).

Efficient Set Research

Another suggestion for future research involves the comparison of efficient sets. Future research will locate the efficient set once with regular ex-post rates of return (or yields) and then compare that with the efficient set once the ex-post data are revised for the probability of default.

Bond Market Efficiency Research

The question of the efficiency of the bond market can be addressed by considering the reaction of the market to a change in the rating assigned by a given bond rating agency.

It is suggested that an analysis of the probability of default calculated before and after a rating change could yield information about (1) the informational content of bond rating change announcements, (2) which rating agency (Moody's or Standard and Poor's) the market follows, and (3) the efficiency of the bond market as reflected in the rapidity with which it adjusts to the change in rating.

Early Warning System Research

Alternatively, it is suggested that the magnitude of delta and perhaps more importantly, changes in delta may serve as an early warning system which may be observed to precede a change in the rating. Additionally, delta can be utilized as an early warning system for regulators who supervise regulated industries such as financial institutions. Note that it is not necessary that the entity studied be publicly traded. Banks, for instance, can be compared one against the other on the basis of deltas calculated from earnings before interest and taxes (EBIT). This approach could serve as the basis for setting FDIC deposit insurance premia for commercial banks, so this research is suggested to have practical applications.

Bond Rating Accuracy Research

The accuracy of bond ratings can be studied by determining whether tnere is a significant difference in the values of delta within a given bond rating category. Delta is suggested as an alternative to bond ratings. As a result, one could calculate deltas for a sample of bonds, rank them by the magnitude of delta, and conduct "goodness of fit" tests on deltas and bond ratings.

Other Research

This paper has abstracted from the implications of differential tax rates. It is suggested that including the tax effect may (and perhaps should) lead to a more individualized formulation of delta unique to investors in a given tax bracket. This result is expected due to the differential tax treatment of income from different types of bonds.

Finally, there is a need to develop the mathematics of the behavior of the probability of default over the length of the investment horizon. The question to be addressed is the manner in which the probability of not defaulting over the investment horizon is derived. Is it equal to the probability of not defaulting in the first subperiod times the probability of not defaulting in the second subperiod given no default in the first period? The mathematical

relationship should be of interest, for it addresses the question of the resolution of default risk over time.

Conclusions

This paper has presented a stochastic dominance technique for quantifying differences in cumulative probability distributions of data. The proposed technique has many potential applications in economic analysis, for it can be used to quantify the remaining dimension along which data differ, all else held constant. The technique was demonstrated on bond data and resulted in the quantifying of the relative probabilities of default between bond rating categories.

The results of the application of the proposed technique were found to be of economic significance. Potential limitations of the research were addressed. This led logically into a discussion of proposals for future research in areas which are closely related but which for various reasons were beyond the scope of this immediate paper. They may serve as the basis for future papers which will serve to round out the research initiated by but by no means completed in this study.

APPENDIX A
A COMPARISON OF MOODY'S AND S&P's CORPORATE BOND RATING DESCRIPTIONS

The following is quoted from Moody's Bond Record, July 1981:

Aaa

Bonds which are rated Aaa are judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edge." Interest payments are protected by a large or by an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.

Aa

Bonds which are rated Aa are judged to be of high quality by all standards. Together with the Aaa group they comprise what are generally known as high grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present which make the long term risks appear somewhat larger than in Aaa securities.

A

Bonds which are rated A possess many favorable investment attributes and are to be considered as upper medium grade obligations. Factors giving security to principal and interest are considered adequate but elements may be present which suggest a susceptibility to impairment sometime in the future.

Baa

Bonds which are rated Baa are considered as medium grade obligations, i.e., they are neither highly protected nor poorly secured. Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and in fact have speculative characteristics as well.

Ba

Bonds which are rated Ba are judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.

B

Bonds which are rated B generally lack characteristics of the desirable investment. Assurance of interest and principal payments or of maintenance of other terms of the contract over any long period of time may be small.

Caa

Bonds which are rated Caa are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.

Ca

Bonds which are rated Ca represent obligations which are speculative in a high degree. Such issues are often in default or have other marked shortcomings.

C

Bonds which are rated C are the lowest rated class of bonds and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

The following is quoted from Standard and Poor's Fixed Income Investor, July 18, 1981, p. 807:

AAA Bonds rated AAA have the highest rating assigned by Standard & Poor's to a debt obligation. Capacity to pay interest and repay principal is extremely strong.

AA These bonds have a very strong capacity to pay interest and repay principal and differ from the highest rated issues only in small degree.

A These bonds have a strong capacity to pay interest and repay principal although they are somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than bonds in higher rated categories.

BB, B, CCC, CC These bonds are regarded, on balance, as predominantly speculative with respect to capacity to pay interest and repay principal in accordance with the terms of the obligation. BB indicates the lowest degree of speculation and CC the highest degree of speculation. While such bonds will likely have some quality and protective characteristics, these are outweighed by large uncertainties or major risk exposures to adverse conditions.

C This rating is reserved for income bonds on which no interest is being paid.

D Bonds rated D are in default, and payment of interest and/or repayment of principal is in arrears.

Plus (+) or Minus (-): The ratings from 'AA' to 'B' may be modified by the addition of a plus or minus sign to show relative standing within the major rating categories.

APPENDIX B
A COMPARISON OF MOODY'S AND S&P'S CORPORATE BOND RATING DETERMINATIONS

The following is quoted from Moody's Bond Record, July 1981:

MOODY'S BOND RATINGS

Purpose: The system of rating securities was originated by John Moody in 1909.

The purpose of Moody's Ratings is to provide the investors with a simple system of gradation by which the relative investment qualities of bonds may be noted.

Rating Symbols: Gradations of investment quality are indicated by rating symbols, each symbol representing a group in which the quality characteristics are broadly the same. There are nine symbols as shown below, from that used to designate least investment risk (i.e., highest investment quality) to that denoting greatest investment risk (i.e., lowest investment quality):

Aaa Aa A Baa Ba B Caa Ca C

Changes in Rating: The quality of most bonds is not fixed and steady over a period of time, but tends to undergo change. For this reason changes in ratings occur so as to reflect these variations in the intrinsic position of individual bonds.

A change in rating may thus occur at any time in the case of an individual issue. Such rating change should serve notice that Moody's observes some alteration in the investment risks of the bond or that the previous rating did not fully reflect the quality of the bond as now seen. While because of their very nature, changes are to be expected more frequently among bonds of lower ratings than among bonds of higher ratings, nevertheless the user of bond ratings should keep close and constant check on all ratings--both high and low ratings--thereby to be able to note promptly any signs of change in investment status which may occur.

Limitations to Uses of Ratings: Bonds carrying the same ratings are not claimed to be of absolutely equal quality. In a broad sense they are alike in position, but since there are only nine rating classes used in grading thousands of bonds, the symbols cannot reflect the fine shading of risks which actually exist. Therefore, it should be evident to the user of ratings that

two bonds identically rated are unlikely to be precisely the same in investment quality.

As ratings are designed exclusively for the purpose of grading bonds according to their investment qualities, they should not be used alone as a basis for investment operations. For example, they have no value in forecasting the direction of future trends of market price. Market price movements in bonds are influenced not only by the quality of individual issues but also by changes in money rates and general economic trends, as well as by the length of maturity, etc. During its life even the best quality bond may have wide price movements, while its high investment status remains unchanged.

The matter of market price has no bearing whatsoever on the determination of ratings which are not to be construed as recommendations with respect to "attractiveness." The attractiveness of a given bond may depend on its yield, its maturity date or other factors for which the investor may search, as well as on its investment quality, the only characteristic to which the rating refers.

Since ratings involve judgments about the future, on the one hand, and since they are used by investors as a means of protection, on the other, the effort is made when assigning ratings to look at "worst" potentialities in the "visible" future, rather than solely at the past record and the status of the present. Therefore, investors using the rating should not, except to find in them a reflection of statistical factors alone, since they are an appraisal of long term risks, including the recognition of many non-statistical factors.

Though ratings may be used by the banking authorities to classify bonds in their bank examination procedure, Moody's Ratings are not made with these bank regulations in view. Moody's Investors Service's own judgment as a desirability or non-desirability of a bond for bank investment purposes is not indicated by Moody's Ratings.

Moody's Ratings represent the mature opinion of Moody's Investors Service, Inc. as to the relative investment classification of bonds. As such, they should be used in conjunction with the description and statistics appearing in Moody's Manuals. Reference should be made to these statements for information

regarding the issuer. Moody's Ratings are not commercial credit ratings. In no case is default or receivership to be imputed unless expressly so stated in the Manual.

The following is quoted from Standard and Poor's Fixed Income Investor, July 18, 1981, p. 807:

S&P ratings are based, in varying degrees, on the following considerations:

1. Likelihood of default--capacity and willingness of the obligor as to the timely payment of interest and repayment of principal in accordance with the terms of the obligation;

2. Nature of and provisions of the obligation;

3. Protection afforded by, and relative position of, the obligation in the event of bankruptcy and other laws affecting creditors' rights.

APPENDIX C
REVIEW OF STOCHASTIC DOMINANCE RULES

Stochastic Dominance Rules: A Review

Stochastic Dominance Criteria[1] provide techniques for ranking alternatives, and have an advantage of being applicable to any distribution analyzed, regardless of its shape, and of requiring very little information about investors' preferences.

Let $F(x)$ and $G(x)$ represent the cumulative probability distributions of the variable x for securities F and G, respectively. An alternative definition of stochastic dominance requires the use of quantiles of the distributions, so we define $Q_F(p)$ and $Q_G(p)$ as the pth quantile of $F(x)$ and $G(x)$, respectively. $Q_F(p)$ is defined as

$$p = Pr_F(x \leq Q_F(p))$$

$Q_G(p)$ is similarly defined.

Finally, the set of all non-decreasing utility functions is denoted by U_1; the set of all concave non-decreasing utility functions by U_2; and the set of all concave non-decreasing utility functions with convex marginal utility by U_3. In terms of stochastic dominance, F is said to dominate G if $E_F U(x) \geq E_G U(x)$ for all $u \in U_{(.)}$, and the strict inequality holds for at least one u_0. The Stochastic Dominance Rules are as follows:

---

[1]For the proof of the Stochastic Dominance Criteria presented, see Quirk and Saposnik (1962), Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970), and Whitmore (1970). Stochastic Dominance Criteria expressed in terms of quantiles are presented in Levy and Kroll (1978).

First Degree Stochastic Dominance (FSD)

A necessary and sufficient condition for F to be said to dominate G (hereafter FDG) for all $u \in U_1$ (i.e., for all u with $u' \geq 0$) is $F(x) \leq G(x)$ for all x, and $F(x_0) < G(x_0)$ for some x_0.

Alternatively, in terms of quantiles, FDG by FSD if $Q_F(p) \geq Q_G(p)$ for all $0 \leq p \leq 1$, and for some p_0, the strict inequality holds.

Second Degree Stochastic Dominance (SSD)

A necessary and sufficient condition for FDG by SSD for all $u \in U_2$ (i.e., for all u with $u' \geq 0$, $u'' \leq 0$) is

$$\int_{-\infty}^{x} [G(t) - F(t)]dt \geq 0 \qquad \text{for all } x$$

and the strict inequality holds for some x_0.

Alternatively, SSD in terms of quantiles is

$$\int_0^p Q_F(t)dt \geq \int_0^p Q_G(t)dt \qquad \text{for all } 0 \leq p \leq 1$$

and the strict inequality holds for some p_0.

Third Degree Stochastic Dominance (TSD)

A necessary and sufficient condition for FDG by TSD for all $u \in U_3$ (i.e., for all u with $u' \geq 0$, $u'' < 0$, $u''' \geq 0$) is

$$\int_{-\infty}^{x}\int_{-\infty}^{t}[G(v) - F(v)]dvdt \geq 0 \qquad \text{for all } x$$

(with the strict inequality holding for some x_0) and

$$\int_{-\infty}^{\infty}[G(t) - F(t)]dt \geq 0$$

Alternatively, in quantiles, TSD is

$$\int_{0}^{p}\int_{0}^{t}[Q_F(z) - Q_G(z)]dzdt \geq 0$$

for $0 \leq p \leq 1$, and the strict inequality holds for some p_0; and

$$\int_{0}^{1}[Q_F(t) - Q_G(t)]dt \geq 0$$

APPENDIX D
SIX MONTH TOTAL HOLDING PERIOD RETURNS
CALCULATED FOR PORTFOLIOS

AAA

	Nominal Returns		Real Returns	
	1st 6 Months	2nd 6 Months	1st 6 Months	2nd 6 Months
1969	0.9858	0.8998	0.9607	0.8737
1970	0.9820	1.1095	0.9528	1.0817
1971	1.0953	1.0465	1.0737	1.0278
1972	1.0315	1.0639	1.0160	1.0462
1973	0.9901	1.0077	0.9604	0.9658
1974	0.9412	1.0189	0.8909	0.9625
1975	1.0736	1.0516	1.0302	1.0144
1976	1.0788	1.0913	1.0534	1.0614
1977	1.0081	0.9976	0.9750	0.9671
1978	1.0169	0.9714	0.9813	0.9277
1979	1.0385	0.9141	0.9869	0.8574
1980	1.0920	0.8313	1.0179	0.7913

AA

	Nominal Returns		Real Returns	
	1st 6 Months	2nd 6 Months	1st 6 Months	2nd 6 Months
1969	0.8910	1.1162	0.8683	1.0838
1970	0.9666	1.0845	0.9378	1.0573
1971	1.0528	1.0999	1.0321	1.0802
1972	1.0254	1.0508	1.0098	1.0333
1973	1.0167	0.9991	0.9862	0.9575
1974	0.8869	0.9861	0.8395	0.9315
1975	1.0880	1.0523	1.0440	1.0150
1976	1.0857	1.1058	1.0602	1.0755
1977	1.0470	1.0106	1.0127	0.9797
1978	1.0126	0.9660	0.9771	0.9225
1979	1.0263	0.9195	0.9753	0.8625
1980	1.0501	0.8788	0.9788	0.8365

A

	Nominal Returns		Real Returns	
	1st 6 Months	2nd 6 Months	1st 6 Months	2nd 6 Months
1969	0.9780	1.0183	0.9531	0.9887
1970	0.9733	1.0728	0.9443	1.0459
1971	1.0580	1.0926	1.0372	1.0731
1972	1.0264	1.0532	1.0108	1.0357
1973	1.0101	1.0042	0.9798	0.9588
1974	0.9417	1.0029	0.8914	0.9474
1975	1.0912	1.0373	1.0471	1.0006
1976	1.0080	1.2585	0.9843	1.2240
1977	1.0562	1.0271	1.0216	0.9957
1978	0.9848	1.0018	0.9503	0.9567
1979	1.0206	0.9268	0.9699	0.8693
1980	1.0557	0.8803	0.9841	0.8379

BAA

	Nominal Returns		Real Returns	
	1st 6 Months	2nd 6 Months	1st 6 Months	2nd 6 Months
1969	0.9613	0.9378	0.9368	0.9106
1970	1.0110	1.0671	0.9809	1.0404
1971	1.0794	1.1371	1.0581	1.1168
1972	1.0591	1.0365	1.0430	1.0193
1973	1.0134	1.0037	0.9830	0.9619
1974	0.9009	0.9779	0.8528	0.9238
1975	1.0664	1.0306	1.0233	0.9941
1976	1.1430	1.1474	1.1161	1.1159
1977	1.0480	1.0319	1.0136	1.0004
1978	1.0085	1.0071	0.9732	0.9618
1979	0.9862	0.9422	0.9372	0.8838
1980	1.0304	0.9091	0.9605	0.8653

APPENDIX E
CHANGES IN UNADJUSTED GNP USED TO CONVERT NOMINAL SIX MONTH TOTAL HOLDING PERIOD RETURNS TO REAL SIX MONTH TOTAL HOLDING PERIOD RETURNS

	First 6 Months	Second 6 Months	Annual
1969	1.0261	1.0299	1.0568
1970	1.0307	1.0257	1.0572
1971	1.0201	1.0182	1.0387
1972	1.0154	1.0169	1.0326
1973	1.0309	1.0434	1.0756
1974	1.0564	1.0586	1.1183
1975	1.0421	1.0367	1.0803
1976	1.0241	1.0282	1.0530
1977	1.0339	1.0315	1.0665
1978	1.0363	1.0471	1.0851
1979	1.0527	1.0661	1.1223
1980	1.0728	1.0506	1.1271

REFERENCES

Altman, E. I. "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," Journal of Finance, September 1968, pp. 589-609.

Altman, E. I. "Railroad Bankruptcy Propensity," Journal of Finance, May 1971, pp. 333-345.

Altman, E. I., R. G. Haldeman and P. Narayan. "ZETA Analysis," Journal of Banking and Finance, June 1977, pp. 29-54.

Beaver, W., P. Kettler and M. Scholes. "The Association Between Market Determined and Accounting Determined Risk Measures," Accounting Review, October 1970, pp. 654-682.

Bierwag, G. and G. Kaufman. "Coping With the Risk of Interest Rate Fluctuations: A Note," Journal of Business, July 1977, pp. 364-370.

Black, F., M. C. Jensen and M. Scholes. "The Capital Asset Pricing Model: Some Empirical Tests." In Studies in the Theory of Capital Markets. M. C. Jensen, ed. New York, N.Y.: Praeger Publishers, 1972, pp. 79-121.

Board of Governors of the Federal Reserve System. Federal Reserve Bulletin, Washington, D.C.

Boardman, C. M., and R. W. McEnally. "Factors Affecting Seasoned Corporate Bond Prices," Journal of Financial and Quantitative Analysis, June 1981, pp. 207-226.

Breen, W. J. and E. M. Lerner. "Corporate Financial Strategies and Market Measures of Risk and Return," Journal of Finance, May 1973, pp. 339-351.

Brigham, E. F. Financial Management: Theory and Practice, 2nd Edition. Hinsdale, Ill.: Dryden Press, 1979.

Cohen, J. B., E. D. Zinbarg and Arthur Zeikel. Investment Analysis and Portfolio Management, 3rd Edition. Homewood, Ill.: Richard D. Irwin, Inc., 1977.

Douglas, G. W. "Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency," Yale Economic Essays, no. 9, Spring 1969.

Fama, E. F. "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance, May 1970, pp. 384-387.

Fisher, I. The Theory of Interest. New York: Macmillan Co., 1930.

Fisher, L. "Determinants of Risk Premiums on Corporate Bonds," Journal of Political Economy, June 1959, pp. 217-237.

Foster, G. Financial Statement Analysis. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1978.

Friend, I. and M. Blume. "Measurement of Portfolio Performance Under Uncertainty," American Economic Review, September 1970, pp. 561-575.

Friend, I., R. Westerfield and M. Granito. "New Evidence on the Capital Asset Pricing Model," Journal of Finance, June 1978, pp. 903-920.

Gonedes, N. J. "Evidence on the Information Content of Accounting Numbers: Accounting-Based and Market-Based Estimates of Systematic Risk," Journal of Financial and Quantitative Analysis, June 1973, pp. 407-443.

Grier, P. and S. Katz. "The Differential Effects of Bond Rating Changes Among Industrial and Public Utility Bonds by Maturity," Journal of Business, April 1976, pp. 226-239.

Hadar, J. and W. R. Russell. "Rules for Ordering Uncertain Prospects," American Economic Review, March 1969, pp. 25-34.

Hanoch, G. and H. Levy. "The Efficiency Analysis of Choices Involving Risk," Review of Economic Studies, July 1969, pp. 335-346.

Hastie, K. L. "Determinants of Municipal Bond Yields," Journal of Financial and Quantitative Analysis, June 1972, pp. 1729-1748.

Hettenhouse, G. and W. Sartoris. "An Analysis of the Informational Value of Bond Rating Changes," Quarterly Review of Economics and Business, Summer 1976, pp. 65-78.

Hickman, W. B. Corporate Bond Quality and Investor Experience. Princeton, N.J.; Princeton University Press, 1958.

Hickman, W. B. Statistical Measures of Corporate Bond Financing Since 1900. Princeton, N.J.: Princeton University Press, 1960.

Hicks, J. R. Value and Capital. Oxford: Clarendon Press, 1939.

Hopewell, M. and G. Kaufman. "Bond Price Volatility and Term to Maturity: A Generalized Respecification," American Economic Review, September 1973, pp. 749-753.

Jensen, M. "The Performance of Mutual Funds in the Period 1945-64," Journal of Finance, May 1968, pp. 389-416.

Kaplan, R. S. and G. Urwitz. "Statistical Models of Bond Ratings: A Methodological Inquiry," Journal of Business, April 1979, pp. 231-261.

Katz, S. "The Price Adjustment Process of Bonds to Rating Reclassifications," Journal of Finance, May 1974, pp. 551-559.

Kaufman, G. "Duration, Planning Period and Tests of the Capital Asset Pricing Model," Journal of Financial Research, Spring 1980, pp. 1-9.

Keynes, J. M. A Treatise on Money. New York, N.Y.: Harcourt, Brace and Company, 1930.

Kroll, Y. "State Contingent Stochastic Dominance," Unpublished Manuscript, 1981.

Levhari, D. and H. Levy. "The Capital Asset Pricing Model and the Investment Horizon," Review of Economics and Statistics, February 1978, pp. 92-104.

Levy, H. "The CAPM and the Investment Horizon," Journal of Portfolio Management, Winter 1981, pp. 32-40.

Levy, H. and Y. Kroll. "Efficiency Analysis with Borrowing and Lending: Criteria and Their Effectiveness," Review of Economics and Statistics, February 1979, pp. 125-130.

Levy, H. and Y. Kroll. "Ordering Uncertain Options With Borrowing and Lending," Journal of Finance, May 1978, pp. 553-574.

Levy, H. and M. Sarnat. Investment and Portfolio Analysis. New York, N.Y.: John Wiley and Sons, Inc., 1972.

Lintner, J. "Security Prices, Risk and Maximal Gains from Diversification," Journal of Finance, December 1965, pp. 587-616.

Logue, D., and L. J. Merville. "Financial Policy and Market Expectations," Financial Management, Summer 1972, pp. 37-44.

Lutz, Friedrich A. "The Structure of Interest Rates," Quarterly Journal of Economics, November 1940, pp. 36-63.

Macaulay, F. R. Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the United States Since 1856. New York, N.Y.: National Bureau of Economic Research, 1938.

McCallum, J. "The Expected Holding Period Return, Uncertainty and the Term Structure of Interest Rates," Journal of Finance, May 1975, pp. 307-323.

McNeil, K., F. J. Kelly and J. T. McNeil. Testing Research Hypotheses Using Multiple Linear Regression. Carbondale and Edwardsville, Ill.: Southern Illinois University Press, 1975.

Miller, H. M. and M. Scholes. "Rates of Return in Relation to Risk: A Re-examination of Some Recent Findings." In Studies in the Theory of Capital Markets, M. C. Jensen, ed. New York, N.Y.: Praeger Publishers, 1972.

Moody Investors Service. Moody's Bond Record, July 1981.

Mossin, Jan. "Equilibrium in a Capital Asset Market," Econometrica, October 1966, pp. 768-783.

National Bureau of Economic Research, Inc. NBER Reporter, Spring 1981.

Percival, J. "Corporate Bonds in a Market Model Context," Journal of Business Research, October 1974, pp. 461-468.

Pinches, G. E. and K. A. Mingo. "A Multivariate Analysis of Industrial Bond Ratings," Journal of Finance, March 1973, pp. 1-18.

Pye, G. "Gauging the Default Premium," Financial Analysts Journal, January-February 1974, pp. 49-52.

Quirk, J. P. and R. Saposnik. "Admissibility and Measurable Utility Functions," Review of Economic Studies, February 1962, pp. 140-146.

Reilly, F. K. Investments. Hinsdale, Ill.: Dryden Press, 1982.

Reilly, F. K. and M. D. Joehnk. "The Association Between Market-Determined Risk Measures and Bond Ratings," Journal of Finance, December 1976, pp. 1387-1403.

Robichek, A. and S. Myers. "Conceptual Problems in the Use of Risk-Adjusted Discount Rates," Journal of Finance, December 1966, pp. 727-730.

Roll, R. "A Critique of the Asset Pricing Theory's Tests: Part I: On the Past and Potential Testability of the Theory," Journal of Financial Economics, March 1977, pp. 129-176.

Roll, R. "Ambiguity when Performance is Measured by the Securities Market Line," Journal of Finance, September 1978, pp. 1051-1069.

Ross, S. A. "The Arbitrage Theory of Capital Asset Pricing," Journal of Economic Theory, December 1976, pp. 341-360.

Rothschild, M. and J. E. Stiglitz. "Increasing Risk: I. A Definition," Journal of Economic Theory, September 1970, pp. 225-243.

Sharpe, W. F. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, September 1964, pp. 425-442.

Sharpe, W. F. Investments, 2nd Edition. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1981.

Sharpe, W. F. "Mutual Fund Performance," Journal of Business, January 1966, pp. 119-138.

Silvers, J. B. "An Alternative to the Yield Spread as a Measure of Risk," Journal of Finance, September 1973, pp. 933-955.

Standard and Poor. Fixed Income Investor, July 1981, p. 807.

Treynor, J. L. "How to Rate Management of Investment Funds," Harvard Business Review, January/February 1965, pp. 63-75.

von Neumann, J. and O. Morgenstern. Theory of Games and Economic Behavior, 3rd Edition. Princeton, N.J.: Princeton University Press, 1953.

Weinstein, M. "The Effect of a Rating Change Announcement on Bond Price," Journal of Financial Economics, December 1977, pp. 329-350.

Weinstein, M. "The Systematic Risk of Corporate Bonds," Journal of Financial and Quantitative Analysis, September 1981, pp. 257-280.

Whitmore, G. A. "Third-Degree Stochastic Dominance," American Economic Review, June 1970, pp. 457-459.

Yawitz, J. B. and W. J. Marshall. "Risk and Return in the Government Bond Market," Journal of Portfolio Management, Summer 1977, pp. 48-52.

BIOGRAPHICAL SKETCH

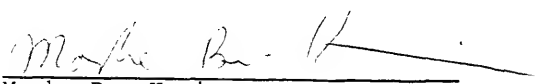
Mrs. Broske, the daughter of George L. and Ruth B. Stearns, was born in Gainesville, Florida, on April 29, 1944. She holds a bachelor's degree from Sweet Briar College, Sweet Briar, Virginia (June, 1966), and a Doctor of Philosophy degree from the University of Florida (August, 1982). She is married to Ernest Cadell Broske, of Richmond, Virginia, and has one child, a daughter, Elizabeth.

Mrs. Broske and her husband and daughter currently reside in South Bend, Indiana, where Mrs. Broske is a member of the faculty of the University of Notre Dame. Mrs. Broske is an Assistant Professor of Finance, and at present teaches a course in investments.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

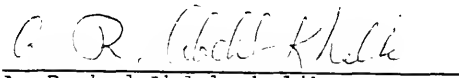
Haim Levy, Chairman
Florida Federal Professor of Finance

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Moshe Ben-Horim
Associate Professor of Finance

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



A. Rashad Abdel-khalik
Graduate Research Professor of Accounting

This dissertation was submitted to the Graduate Faculty of the Department of Finance, Insurance and Real Estate in the College of Business Administration and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1982

Dean for Graduate Studies and Research

UNIVERSITY OF FLORIDA
3 1262 08553 6943